Heat Transfer

Exchange of thermal energy due to a difference in temperature.

Thermodynamics

1st law

$Q = \Delta U + W$. → ~~f~~

There is thermal energy & microwave energy. Here we see thermal energy.

Here W ~~is de is deper~~
- ↳ Const process
- ↳ polytropic
- ↳ spring work
- ↳ electrical work.

ΔU depends on temp. & pressure.

Three modes of heat transfer
1. Conduction
2. Convection
3. Radiation.

Conduction HT

Transfer of thermal energy from more energetic to less energetic particles due to their ~~interaction~~ interaction.

(Heat one side of plate, particles vibrate & interact with neighbours exchanging energy & energy exchange goes from hot

Side of steel plate to cold side by conduction.

Fourier's Law of Heat Conduction

$$q_x'' = -K \frac{dT}{dx}$$

$q \to W$

$q'' = W/m^2$

$q \Rightarrow$ heat rate.

$q'' \Rightarrow W/m^2$.

$q \to W \to$ heat rate or heat transfer

$q'' = W/m^2 \to$ heat flux

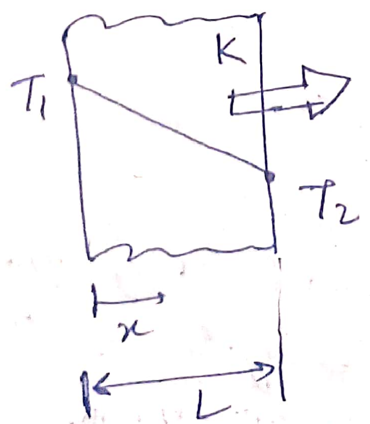
Eqn is for 1D heat transfer case.

K is property called "Thermal Conductivity"

$K \to W/m\cdot K$

(K depend on material & temperature)

1D Heat Transfer



For 1D

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L - 0}$$

Temperature profile is linear in this case.

$$q_x'' = -K\left(\frac{T_2 - T_1}{L - 0}\right)$$

$$= K\left(\frac{T_1 - T_2}{L}\right) = K\frac{\Delta T}{L}$$

From fourier law, for 1D

$$\boxed{q_x'' = K\frac{\Delta T}{L}}$$

$$q_x = K A \frac{\Delta T}{L}$$

A = Area normal to heat flow.

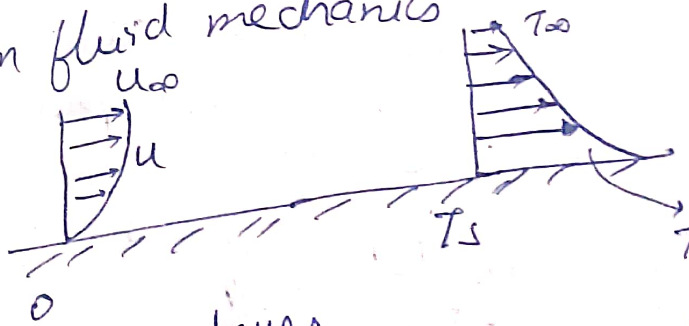
The eq^n is a experimental result.

Heat flows from hot to cold.

'–' sign to make the formula correct i.e. heat flow from hot to cold.

Convection Heat Tranfer (b/w surface & fluid)

from fluid mechanics



Thermal boundary layer.

boundary layer.

$T_s \rightarrow$ surface temp.

$T_\infty \rightarrow$ temp. of flowing air. (it is cooler)

Rate eqn is newtons law of cooling

$$q = hA(T_s - T_\infty)$$

A ⇒ surface area in contact with fluid.

h → convection heat transfer coefficient.

h depends on →

- fluid property
- geometry
- flow regime.

for gases
$h = 25$ to 250 W/m^2K

liquid $h = 100$ to $20,000$ W/m^2K.

Radiation heat transfer

It can occur in vaccum.

The eqn is Stefan-Boltzmann Law

$$E_b = \sigma T_s^4$$ (E = emissive power)

σ = stefan boltzman constant.

$= 5.67 \times 10^{-8}$ W/m^2K^4

It is the maximum energy a surface can emit. It is for ideal emitter called black body or black surface.

For non-black bodies

$$\boxed{E = \varepsilon \sigma T_s^4}$$ → (Just by surface itself)

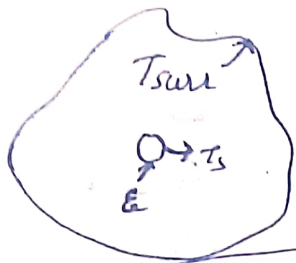
ε = surface emissivity. (E = emissive power)

$$0 \le \varepsilon \le 1$$

If $\varepsilon = 1$, it behaves like a black body.

Radiation heat transfer b/w a small object and ~~walls~~ the walls of a large enclosure

(Temperature is in absolute)



$$\boxed{q'' = \varepsilon \sigma (T_s^4 - T_{surr}^4)}$$

Only valid if there is a small object in a large enclosure.

Lec-2

For radiation

$$q = \varepsilon \sigma A (T_s^4 - T_{surr}^4)$$

absorbed incident radiation

$$G_{abs} = \alpha G$$

G = irradiation.

α = absorbtivity

$$0 \le \alpha \le 1$$

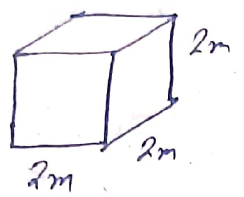
G = irradiation = energy that comes in from surrounding

G (W/m^2)

$q'' = q/A$ = heat flux.

Example

a cubical freezer compartment is 2m on a side



K of styrofoam insulation $K = 0.03$ W/mK
Head load less than 500W.
(inside) $T_i = -10°C$ & $T_o = 35°C$
Find ~~insliu~~ insulation thickness.

$$q = KA\frac{\Delta T}{L}$$

L is thickness of insulation.

$$L = \frac{KA\Delta T}{q}$$

$$= \cancel{\frac{0.03 \times (2\times2) \times (45)}{800}}$$

(bottom is perfectly insulated)
heat comes from top & 4 side walls
$= 5 \times (2 \times 2)$

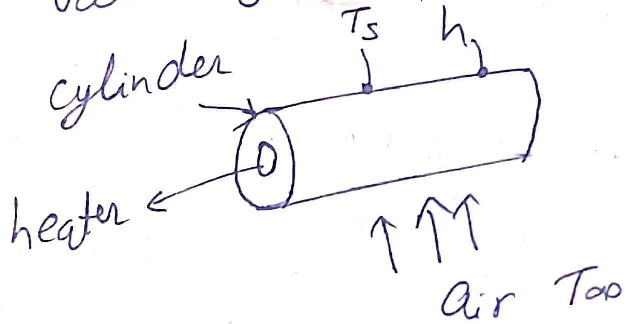
$$L = \frac{0.03 \times 5\times(2\times2)\times(35-(-10))}{500}$$

$$L = \frac{0.03 \times 20 \times 45.9}{\cancel{500}\ \cancel{10}\ 5} = \frac{0.27}{5}$$

$$= 0.054\ m = 5.4\ cm\ (2\ inches)$$

Example

Electric resistance heater embedded in a long cylinder of dia 30 mm. Air at 23°C blows over the cylinder, surface temp 90°C.
Heater puts ~~out~~ out 400 W/m.
Value of h?



$(q = W,\ q' = W/m,\ q'' = W/m^2)$

$$q = hA(T_s - T_\infty) = h \times \pi D L (T_s - T_\infty)$$

$$h = \frac{q}{A(T_s - T_\infty)}$$

$$q' = q/L$$

$$h = \frac{q'}{\pi D (T_s - T_\infty)} = \frac{400\ W/m}{\pi \times 0.03 \times (90 - 23)}$$

$$= \frac{400}{\pi \times 0.03 \times 67} = 65\ W/m^2K$$

Example

Spherical instrumentation package is dia of 100 mm with $\varepsilon = 0.23$ in ~~sta~~ a large space simulation chamber whose walls are at ~~77 K~~ 77 K. The outside temp of package is 40°C. How much power is being dissipated by package.

⇒ We want Q (power)

There, is word large, so we can use radiation eqⁿ.

$$q = \varepsilon \sigma A (T_s^4 - T_{surr}^4)$$

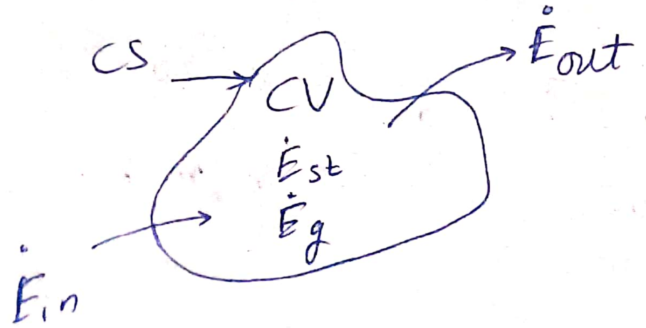
$$= 0.23 \times 5.67 \times 10^{-8} \times 4\pi r^2 \times ((40+273)^4 - (77)^4)$$

$$(4\pi r^2 = \pi D^2)$$

$$= 0.23 \times 5.67 \times 10^{-8} \times \pi \times (0.1)^2 \times (313^4 - 77^4)$$

$$= \cancel{4.3\,W}\; 3.9\,W$$

Energy balance on a control volume



$\dot{E}_{in}$ = rate at which thermal energy enters

$\dot{E}_{out}$ = rate at which thermal energy leaves

$\dot{E}_{st}$ = rate at which thermal energy is stored

$\dot{E}_{g}$ = rate at which thermal energy is generated.

Energy balance:

$$\boxed{\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g} = \dot{E}_{storage}}$$

$\dot{E}$ = energy per time = J/s = W.
(Joules/second)

Consider a surface of the CV.

$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g} = \dot{E}_{st}$,
for a surface.

When thickness approaches 0 for a volume it is a surface.

for a surface

vol $\rightarrow$ 0

(mass = density x volume)

mass $\rightarrow$ 0.

(if no mass $\dot{E}_{storage} \rightarrow 0$

You need mass to generate energy so,

$\dot{E}_{g} \rightarrow 0$.

$\therefore$ $\dot{E}_{in} = \dot{E}_{out}$

So energy balance on a surface of an object is

$$\dot{E}_{in} = \dot{E}_{out}.$$

as volume as mass approach 0.

Lec 3

CV energy balance

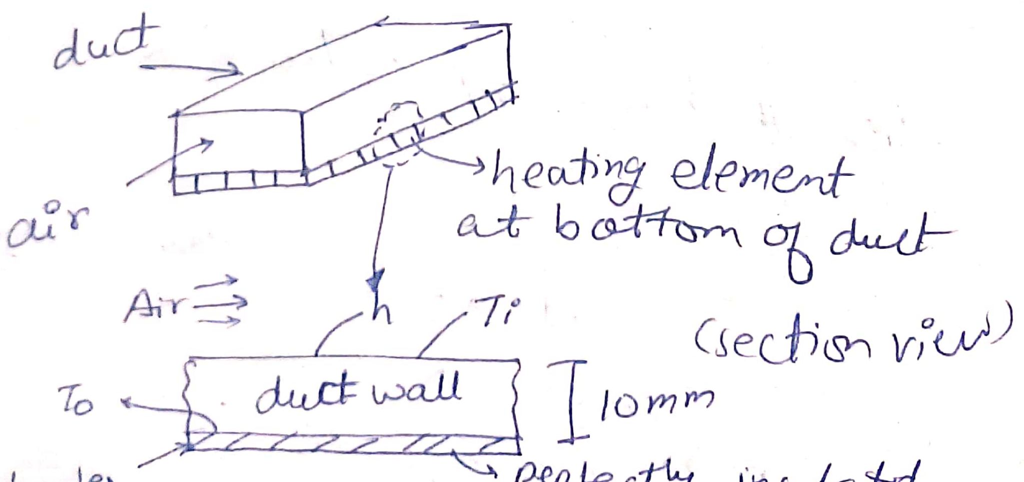
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{out}$$

surface energy balance

$$\dot{E}_{in} = \dot{E}_{out}.$$

Example

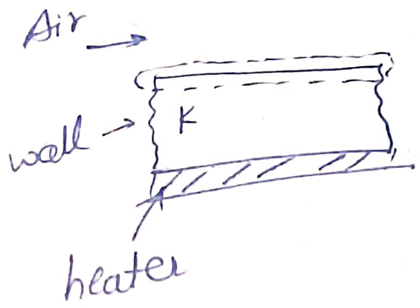
A thin electrical heating element provides a uniform heat flux q_o'' to the bottom of a rectangular duct



$q_o'' = ?$

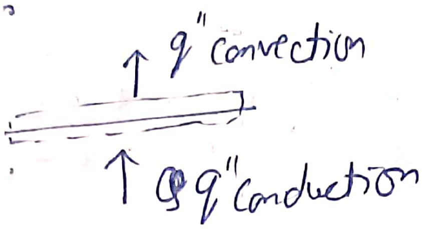
K of duct wall = 20 W/mK | $T_\infty = 30^\circ C$

h = 100 W/m²K | $T_i = 85^\circ C$



Surface CV for upper surface of wall.

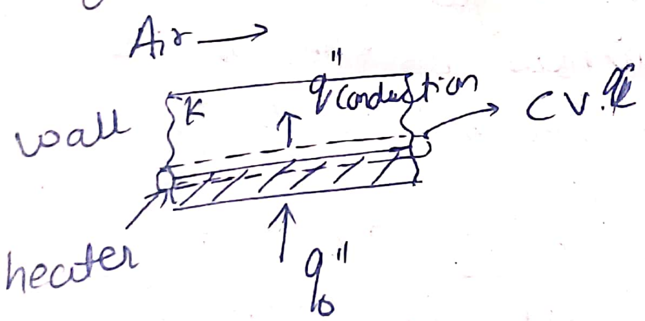
For our control surface we have conduction coming in and convection going out.



$$q''_{cond} = q''_{conv}$$

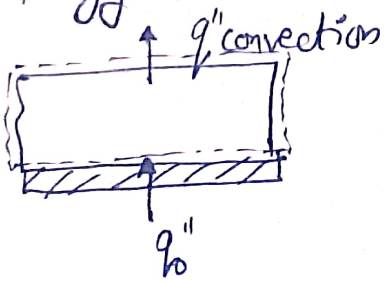
$$\frac{k(T_0 - T_i)}{L} = h(T_i - T_{\infty})$$

Surface energy balance for lower surface



$$q''_0 = q''_{conduction} = \frac{k(T_{\infty} - T_i)}{L}$$

Energy balance on whole wall



No energy generates neither it is stored, so

$$\dot{E}_{in} = \dot{E}_{out}$$

$$q''_0 = q''_{convection}.$$

$$q_o'' = h(T_{si} - T_\infty)$$

$$q_o'' = 100(85 - 30) = 100 \times 55$$

$$= 5500 \text{ W.}$$

Chapter-2

$$\hat{q}'' = -k\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right) = -K\nabla T$$

look at K in more detail.

Solids

Transport of thermal energy

(1) lattice vibrations

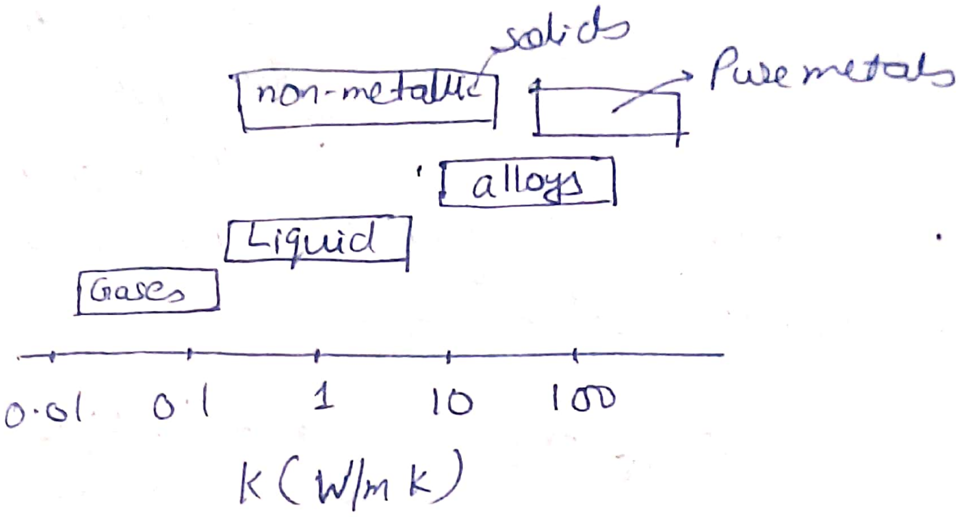
(2) free electron migration

typically in good electrical conductors, so they are also good thermal conductors.

Liquid Thermal energy transport by Kinetic energy collisions of molecules (they have strong bonds)

Gases

same as above but they have weak bonds.

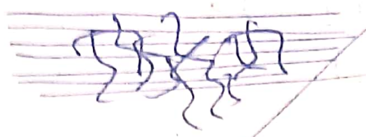


Gases → 0.02 to 0.3 W/mK
Liquid → 0.2 to 8 W/mK
Alloys → 11 to 150 W/mK
Pure metals → 50 to 500 W/mK.
Non-metallic solids → 0.3 to 50 W/mK.
Copper → 400 W/mK
Aluminium → 240 W/mK
Carbon steel → 60 W/mK
Stainless steel → 15 W/mK
Glass → 1.4 W/mK.
water → 0.6 W/mK (Not moving, still)
oil → 0.15 W/mK
air → 0.025 W/mK.

If (air/water are not moving b/w two surfaces then it is a pretty good insulation.
If it is moving then there will be convection.

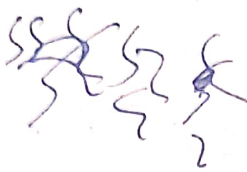
Glass (1.4 W/mK)
Air (0.025 W/mK) > Combine these two glass fiber 0.040 W/mK

We have glass fiber insulation by combining glass & air.



glass fiber layers, air will be b/w them.

glass fiber



glass fiber

air trapped b/w glass fiber

You don't water. inside that, as ~~~~ water will destroy insulation.

Thermal Diffusivity, α

$$\alpha = \frac{K}{\rho C_p} = \frac{\text{rate of heat flow into material}}{\text{ease of stored energy}}$$

aluminium $\alpha = 7 \times 10^{-5}\ m^2/s$

Concrete $\alpha = 0.05 \times 10^{-5}\ m^2/s$.

Thermal conductivity measures how well a material transfers heat, while thermal diffusivity measures how fast heat spreads.

Lec-4

Property tables are described as →

	ρ	C_p	K	$\alpha \cdot 10^6$	K as a fn of T
alum				97.1	
copper					
etc					

at 300 K

$\boxed{\text{Table A1}}$ metallics

$\boxed{\text{Table A2}}$ Non-metallics

Format is similar to table A:1.

$\boxed{\text{Table A3}}$

	ρ	C_p	K
Material			

all values at 300 K.

First part – Structural building materials
Second " – Insulation
Third " – Industrial Insulation
Fourth " – others

$\boxed{\text{Table A4}}$ Gases

ρ	c_p (J/kgK)	μ	ν	k	α

air
H_2
N_2
etc

$\mu \to$ absolute viscosity
$\nu \to$ kinematic viscosity

$\boxed{\text{Table A5}}$ Liquid

ρ	c_p (KJ/kgK)	μ	ν	k	α

engine oil
to
etc

(Units may vary from table to table)

$\boxed{\text{Table A6}}$ Saturated water

T	v_f	v_g	u_f	u_g	h_f	h_g
to						

(temp)

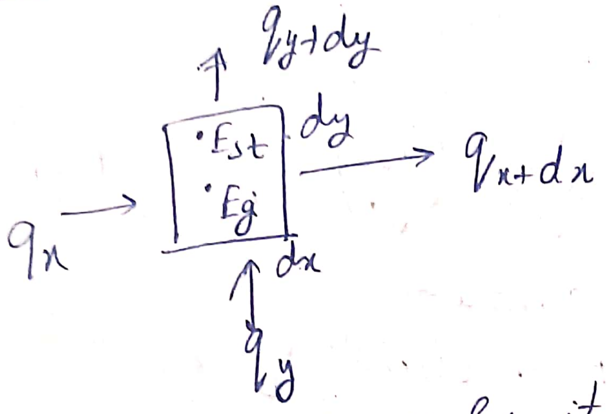
$v \Rightarrow$ specific volume

$\rho = 1/v$

For saturated water

T	ρ	c_p	$\mu \times 10^6$	$\nu \times 10^7$	K	$\alpha \times 10^6$	P_r
$\downarrow$							

The Heat Diffusion Eqⁿ



z is out of book & unit length

Energy balance on the element:

$$q_x = -K \underbrace{(dy)(1)}_{A} \frac{dT}{dx} \quad \& \quad q_y = -K \underbrace{(dx)(1)}_{A} \frac{dT}{dy}$$

and $q_{x+dx} = q_x + \left(\frac{\partial}{\partial x} q_x\right) dx$

$$q_{y+dy} = q_y + \left(\frac{\partial}{\partial y} q_y\right) dy.$$

$$\dot{E}_g = \dot{q} \underbrace{(dx)(dy)(1)}_{} \rightarrow \text{volume}$$

$\dot{q}$ = energy generated per unit volume (W/m^3)

$$\dot{E}_{st} = m c_p \frac{\partial T}{\partial t} \quad \text{(storage energy)}$$

Energy balance for differential CV

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$q_x + q_y - q_{x+dx} - q_{y+dy} + \dot{E}_g = \dot{E}_{st}$$

Substituting gives

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Let K = constant.

Take K out & divide by K.

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}}{K} = \frac{\rho c_p}{K}\frac{\partial T}{\partial t}}$$

2D form of heat diffusion eqⁿ.

but $\alpha = K/\rho c_p$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \dot{q}/K = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If steady state, & no heat generated

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Laplace eqⁿ.

Objective is to find $T(x, y, t)$

We will need initial & boundary condition

For the initial condition (IC)

$$T(x, y, t=0) = \cancel{T_s}\ T_i$$

For the boundary condition (BC)

(1) Constant surface temp.

$$T(x=0, t) = T_s$$

(2) Constant surface heat flux., q_s''

(eg:- a wire wound around a pipe delivers uniform electrical resistance heat to the surface)

$$q_s'' = -K \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

(eg: heater mounted to a surface)

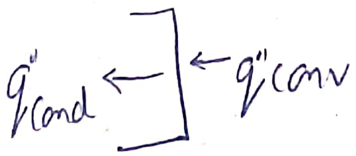
(3) Perfectly insulated, $q_s'' = 0$

(no energy passes)

(symmetry planes)

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \quad \text{(Adiabatic)}$$

(4) Convection on boundary.



$$q''_{convection} = q''_{conduction}$$

$$h(T_{x=0} - T_\infty) = -K \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

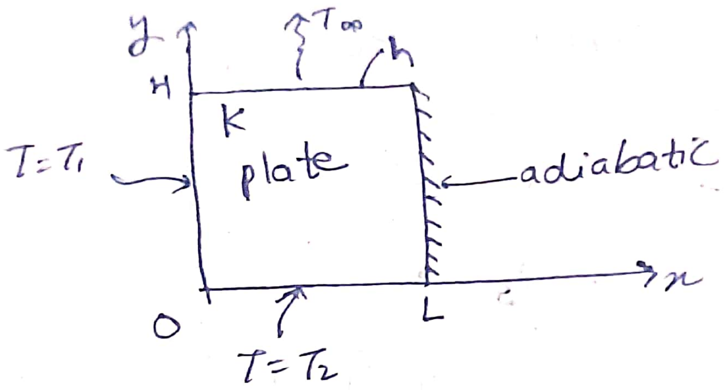
We need one initial condition for time.

We have $\frac{\partial^2 T}{\partial x^2}$, so we need 2 boundary conditions for x & similarly 2 for y.

We need 4 BCs & 1 ICs to solve our governing eq^n & we get $T(x, y, t)$.

Lec-5

Example



- Steady state
- No heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \cancel{\frac{\dot{q}}{K}}^{\,0} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{\,0}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

We need 2 BCs relating temp to x &
2 BCs relating temp to y.

$BC^{\#1}$ $\quad T(x=0, y) = T_1$

$BC^{\#2}$ $\quad T(x, y=0) = T_2$

$BC^{\#3}$ $\quad \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0$

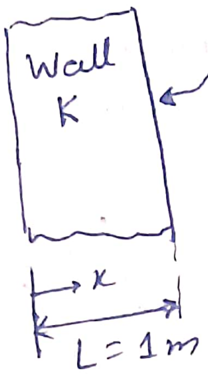
$BC^{\#4}$ Convection

$$-K\left.\frac{\partial T}{\partial y}\right|_{y=H} = h\,(T_{y=H} - T_\infty).$$

So, this problem is properly posed.
Could solve to get $T(x, y)$

First we find temperature distribution then we fourier law to find heat transfer.

Example



$\rho = 1600 \ Kg/m^3$

$K = 40 \ W/mK$

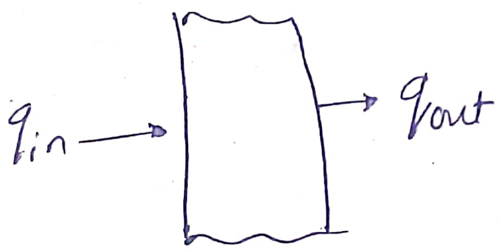
$C_p = 4000 \ J/Kg\,K$

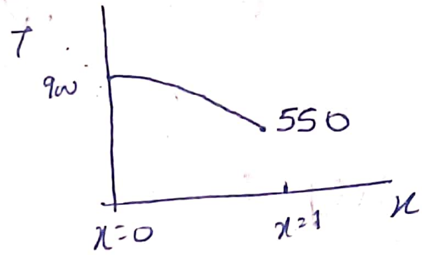
$\dot{q} = 1000 \ W/m^3$

$T(°C) = 900 - 300x - 50x^2$

① Find q into and out of the wall.
② Find rate of change of energy storage in the wall.

⇒





$$T = 900 - 300x - 50x^2$$

$$\frac{\partial T}{\partial x} = -300 - 100x$$

$$q_{in} = -KA\frac{\partial T}{\partial x}\bigg|_{x=0} = -40\times 10\times(-300) = 120 \ KW.$$

So, our assumed dirⁿ is correct

$$\dot{q}_{out} = -KA\frac{\partial T}{\partial x}\bigg|_{x=L} = -40\times 10\times(-400) = 160 \ KW.$$

(b) Energy balance on wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{storage}$$

$$\dot{E}_{gen} = 1000 \times 10 \times 1 = 10,000 \text{ W}$$

$$1,20,000 - 1,60,000 + 10,000 = \dot{E}_{storage}$$

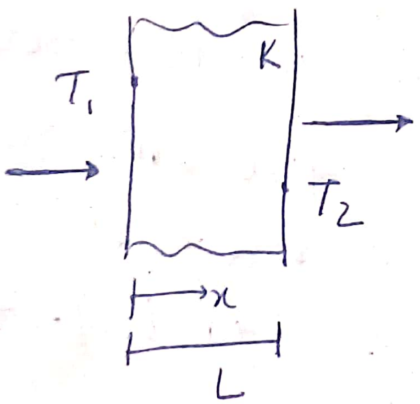
$$-30,000 = \dot{E}_{storage}$$

$\dot{E}_{storage} = \rho C_p \frac{\partial T}{\partial t}$, can also solve for storage energy.

Chapter-3

Assume-

1D, steadystate, constant K, no heat generation



$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\cancel{\frac{\partial^2 T}{\partial y^2}}}_{0\ (1D)} + \underbrace{\cancel{\frac{\dot{q}}{K}}}_{0\ (\text{no gen})} = \underbrace{\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}}_{0\ (\text{Steady state})}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

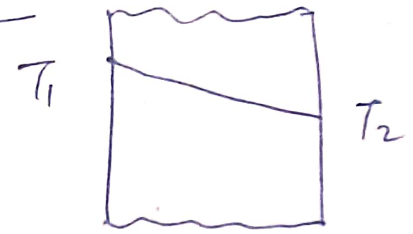
eq^n is $\frac{\partial^2 T}{\partial x^2} = 0$

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) = 0$$

So, $\frac{dT}{dx} = C_1$

$$T(x) = C_1 x + C_2.$$

With all assumption temp variation is linear.



$BC^{\#1}$ $T(x=0) = T_1$

So, $T_1 = 0 + C_2 \Rightarrow C_2 = T_1$

$BC^{\#2}$ $T(x=L) = T_2$

$$T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$$

$$\boxed{T(x) = \frac{T_2 - T_1}{L} x + T_1}$$

Heat Transfer : $q = -KA \frac{dT}{dx}$

$\Rightarrow$ $q = -KA\left(\frac{T_2 - T_1}{L}\right) \Rightarrow \boxed{q = KA\left(\frac{T_1 - T_2}{L}\right)}$

Rewrite to get

(heat flow) $q = \dfrac{T_1 - T_2}{(L/KA)}$ looks similar to

$i = \dfrac{\Delta V}{R}$ (current flow)

$i \rightarrow$ flows difference in voltage

q flows difference in temperature

Thermal resistance $= \dfrac{L}{KA}$

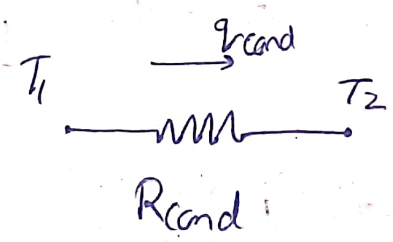
$$\boxed{R_{cond} = \frac{L}{KA}}$$

Higher resistance, so less heat flow.

$L \uparrow \quad q \downarrow$

$A \uparrow \quad q \uparrow$

$K \uparrow \quad q \uparrow$



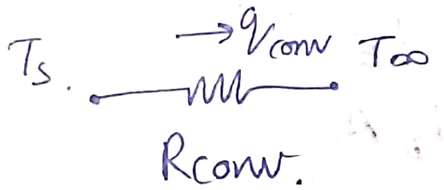
(model of heat flow through a plain wall)

Now, for convection

$$q = hA(T_s - T_\infty) \quad \text{(No assumptions)}$$

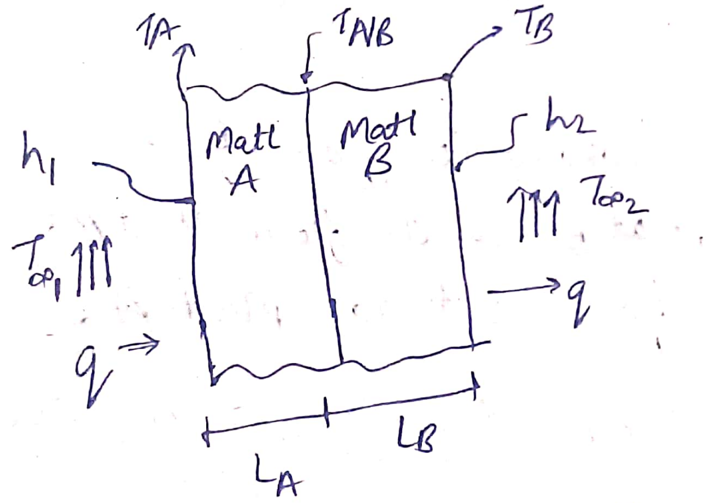
$$q = \frac{(T_s - T_\infty)}{(1/hA)}$$

$$\boxed{R_{conv} = \frac{1}{hA}}$$



Lec-6

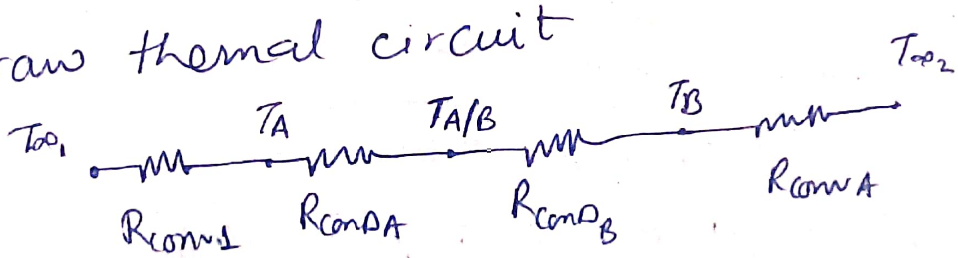
Composite wall (Example)



Assume $T_{\infty_1} > T_{\infty_2}$

wall area $A_A = A_B = A$

Draw thermal circuit



This is a series connection

$$R_{conv_1} = \frac{1}{h_1 A}$$

$$R_{conv_2} = \frac{1}{h_2 A}$$

$$R_{cond_A} = \frac{L_A}{K_A A}$$

$$R_{cond_B} = \frac{L_B}{k_B A}$$

$$q = \frac{(T_{\infty_1} - T_{\infty_2})}{R_{conv_1} + R_{cond A} + R_{cond B} + R_{conv 2}}$$

So, we get heat flow through composite wall.

q is same when it flows from Mat A to Mat B and from Mat B to out and from out to Mat A.

Now to get $T_{A/B}$ (interface temperature)

$$q = \frac{T_{\infty_1} - T_{A/B}}{R_{conv 1} + R_{cond A}}$$ (use q we got earlier)

$$q = \frac{T_{A/B} - T_{\infty 2}}{R_{cond B} + R_{conv 2}}$$

We equate q & get $T_{A/B}$.

We need hottest temperature in A.
It will be T_A. ~~(i.e outside)~~

$$q = \frac{T_{\infty} - T_A}{R_{conv1}}$$

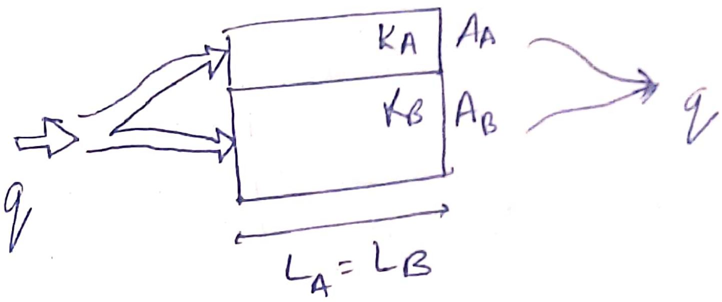
Solve for T_A.

Similarly we can get T_B.

$$q = \frac{T_B - T_{\infty 2}}{R_{conv2}} = \frac{T_{A/B} - T_B}{R_{cond B}}$$

Solve for T_B from any eq^n.

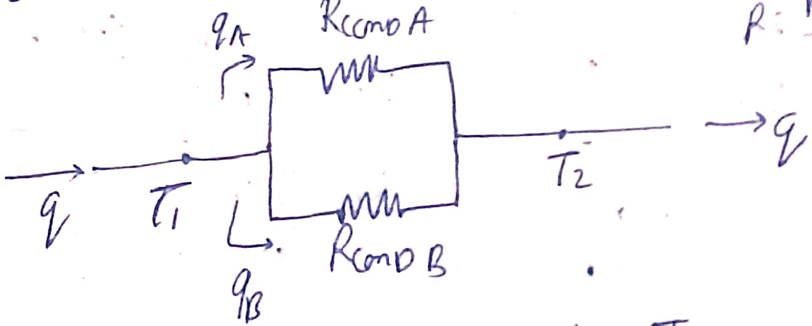
Parallel walls



Heat gets distributed in A & B.
Draw the thermal circuit.

$\left(\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B}\right)$

$R = \frac{R_A R_B}{R_A + R_B}$



$$q = \frac{T_1 - T_2}{R_{eq}} = \frac{T_1 - T_2}{\frac{R_{cond A} * R_{cond B}}{R_{cond A} + R_{cond B}}}$$

$$R_{cond,A} = \frac{L}{K_A A_A}$$

$$R_{cond,B} = \frac{L}{K_B A_B}$$

$$\boxed{q = \frac{T_1 - T_2}{\dfrac{R_{cond A} \times R_{cond B}}{R_{cond A} + R_{cond B}}}}$$

and

$$\boxed{q_A = q\left(\frac{R_{cond B}}{R_{cond,A} + R_{cond,B}}\right)}$$

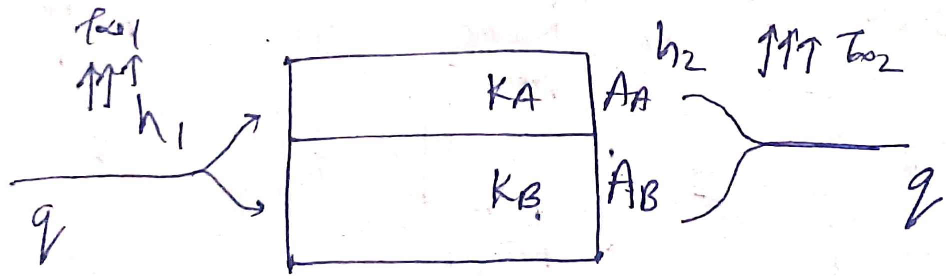
and

$$q_B = q - q_A$$

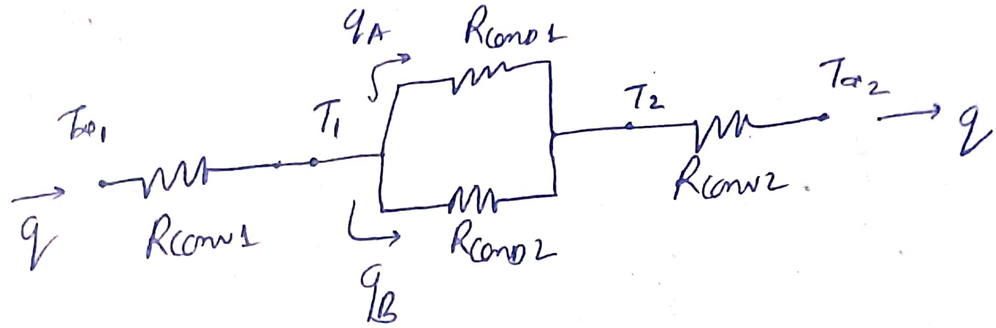
For 3 walls

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}}$$

Now, we include convection.



So, we add resistance for convection.



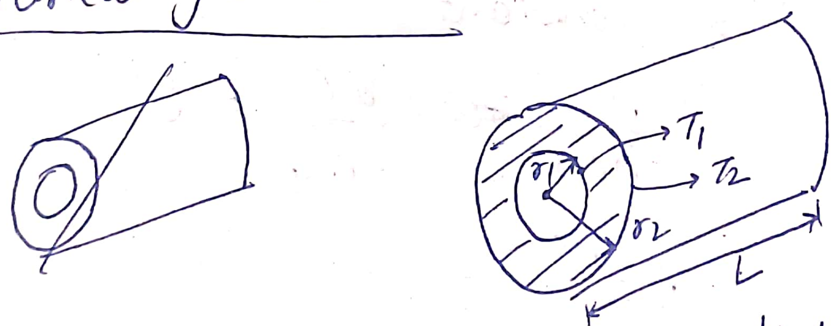
This is a series parallel combination.

$$\boxed{q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv\,1} + \dfrac{R_{cond\,A} \times R_{cond\,B}}{R_{cond\,A} + R_{cond,B}} + R_{conv\,2}}}$$

and $$q_A = q\left(\frac{R_{cond,B}}{R_{cond,B} + R_{cond,A}}\right)$$

and $$q_B = q - q_A$$

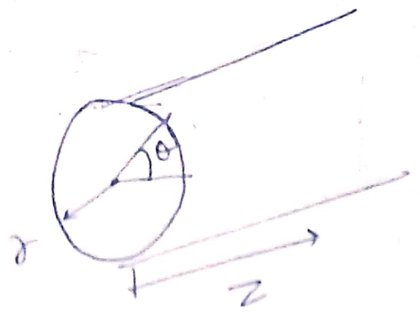
Cylindrical Geometries



How much heat flows from hot temp T_1 to cold temp. T_2.

Cylindrical form of the heat diffusion equation.

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(k r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}}$$



$$\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(K\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)$$
$$+\dot{q}=\rho C_p\frac{\partial T}{\partial t}.$$

For 1D heat transfer

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

For steady state, & no heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right)=0$$

If K = Constant

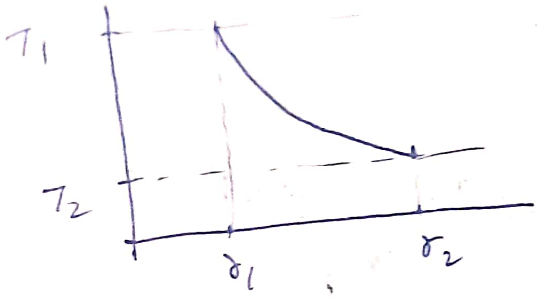
$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0$$

$$r\frac{dT}{dr}=C_1$$

$$\cancel{\frac{dT}{dr}=\frac{C_1}{r}} \quad \frac{dT}{dr}=\frac{C_1}{r}$$

$$\boxed{T(r)=C_1\ln r+C_2}$$

So, logarithmic variation.



(For cylindrical surface)

For plane wall, we had linear variation.

To get C_1 & C_2, apply BCs.

$BC^{\#1}$ $\quad T(r=r_1)=T_1$

$BC^{\#2}$ $\quad T(r=r_2)=T_2$

gives $\boxed{T(r)=\dfrac{T_1-T_2}{\ln(r_2/r_1)}\ln(r/r_2)+T_2}$

Now, we need q.

$$\boxed{\begin{aligned} q &= -KA\frac{dT}{dr} \\ q &= \frac{2\pi KL}{\ln(r_2/r_1)}(T_1-T_2) \end{aligned}}$$

Rewrite to get

$$q=\frac{(T_1-T_2)}{\left(\dfrac{\ln(r_2/r_1)}{2\pi KL}\right)}$$

L = length of tube

$$\boxed{R_{cond.pipe}=\frac{\ln r_2/r_1}{2\pi KL}}$$

Lec-7

Conduction Resistance

Rt Plane wall

$R = \frac{L}{KA}$

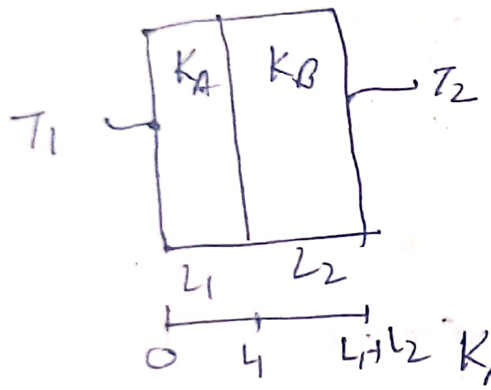
Cylindrical wall

$R = \frac{\ln(r_2/r_1)}{2\pi K L}$

Convection Resistance

$R = \frac{1}{hA}$ (All units are °C/W).

Composite wall

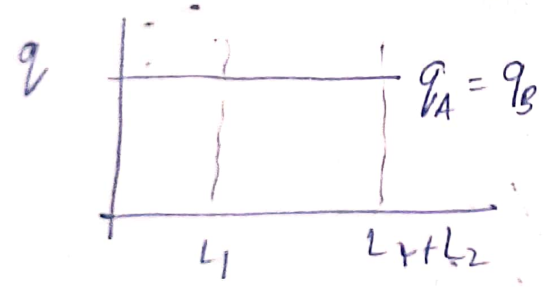


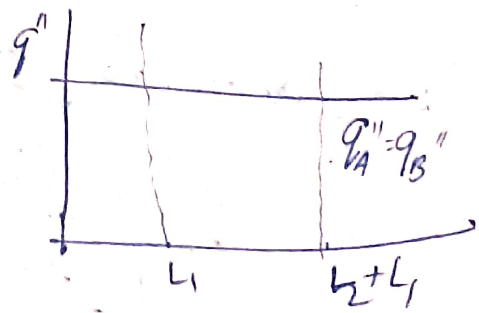
if $K_A = 2K_B$

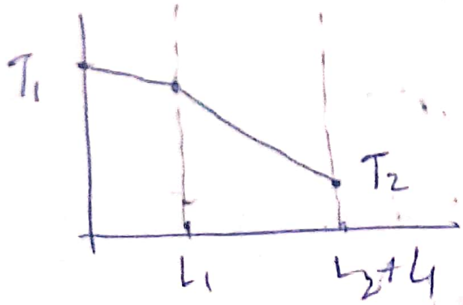
$q_A = q_B$

$q_A'' = q_B''$

$K_A \frac{dT}{dx}\Big|_A = K_B \frac{dT}{dx}\Big|_B$







$K_A \frac{dT}{dx}\Big|_A = K_B \frac{dT}{dx}\Big|_B$

$K_A > K_B$

So, $\frac{dT}{dx}\Big|_A < \frac{dT}{dx}\Big|_B$

Pipe on Tube

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}}$$

$$q = U_i A_i (T_{\infty i} - T_{\infty o}) = U_o A_o (T_{\infty i} - T_{\infty o})$$

$$\text{so, } UA = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}}$$

U = overall heat transfer coefficient

UA = "UA product"

$$q = UA(T_{\infty i} - T_{\infty o})$$

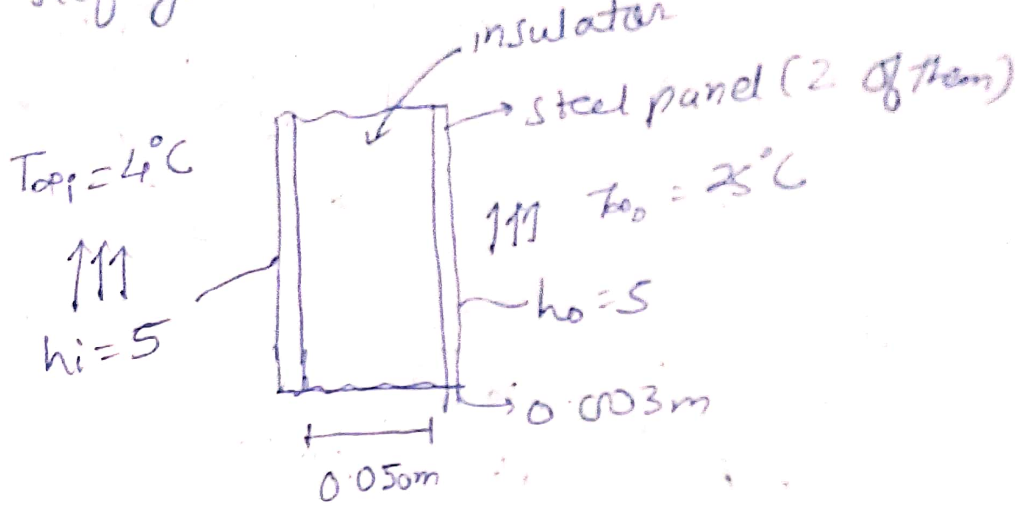
Similar to $q = hA(T_s - T_\infty)$

Based on inside area $U_i A_i$.

Based on outside area $U_o A_o$.

Example

Refrigerator



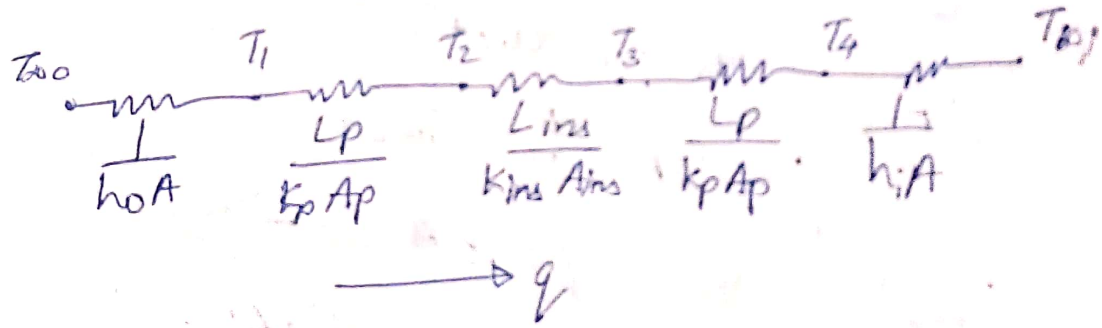
$K_{insulation} = 0.046$ W/mK

$K_{steel} = 60$ W/mK.

Find heat gain per unit area q'' (heat gain)

⇒ Thermal circuit

We start from hot side



$$q = \frac{T_{\infty,o} - T_{\infty;i}}{\frac{1}{h_o A} + \frac{L_p}{K_p A_p} + \frac{L_{ins}}{K_{ins} A_{ins}} + \frac{L_p}{K_p A_p} + \frac{1}{h_i A}}$$

$$q'' = \frac{q}{A} = \frac{T_{\infty,o} - T_{\infty i}}{\frac{1}{h_o} + \frac{L_p}{K_p} + \frac{L_{ins}}{K_{ins}} + \frac{L_p}{K_p} + \frac{1}{h_i}}$$

$$q'' = \frac{25 - 4}{0.2 + 0.00005 + 1.086 + 0.00005 + 0.2}$$

$= 14.1 \ W/m^2.$

High resistance cuts down on heat transfer.
The dominant here is insulation,
There is no resistance from steel panel.

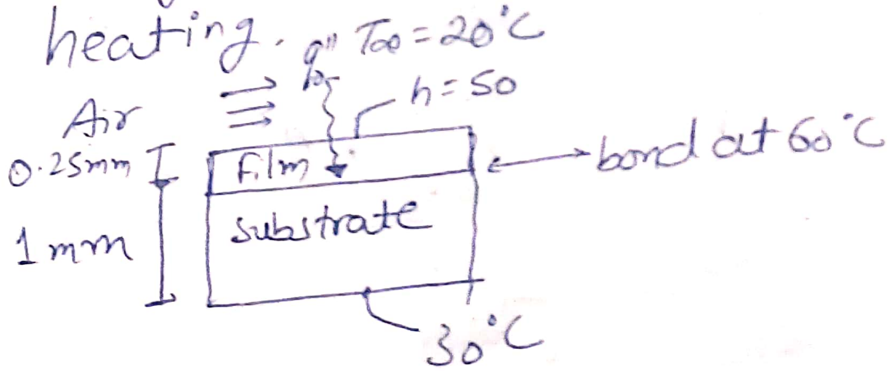
To get T_4

$$q'' = \frac{T_4 - T_{\infty i}}{1/h_i}$$

gives $T_4 = 6.82°C$
Similarly we can ~~get~~ get temperatures.

Example

Curing a transparent film by radiant heating.



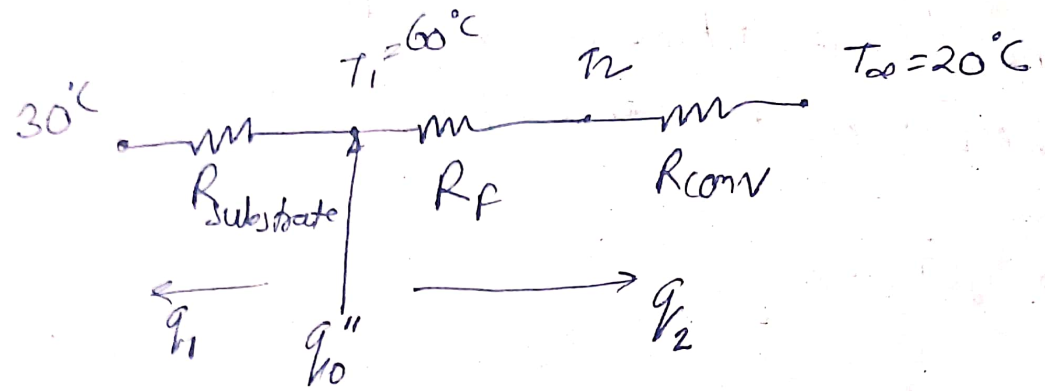
$K_F = 0.025 \ W/mK$

$K_s = 0.05 \ W/mK$

heat goes from film is absorbed at the bonding line. Bonds at 60°C
~~Due~~ All heat flux is absorbed at bonding surface

What heat flux do we need to bind at 60°c

=> We draw thermal circuit.



Heat comes at boriding surface, some goes down, some goes upto the air.

$$q_0'' = q_1'' + q_2'' \quad (\text{Same Area})$$

$$= \frac{60-30}{\frac{L_s}{k_s}} + \frac{60-20}{\frac{L_f}{k_f} + \frac{1}{hA}}$$

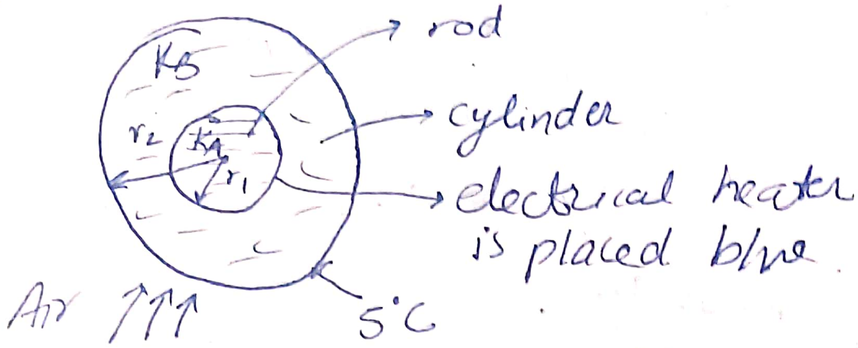
$$q_0'' = \frac{30}{\frac{0.001}{0.05}} + \frac{40}{\frac{0.00025}{0.025} + \frac{1}{50}}$$

$$= 1500 + 1333.33$$

$$= 2833.33 \ W/m^2$$

We can also find T_2.

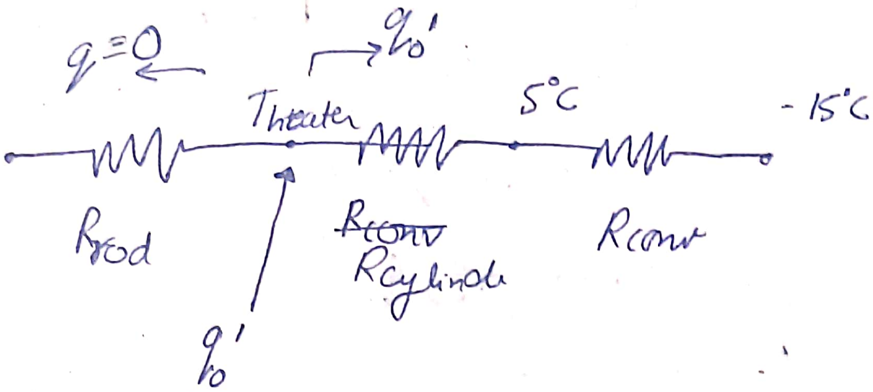
Example



$T_\infty = -15°C$

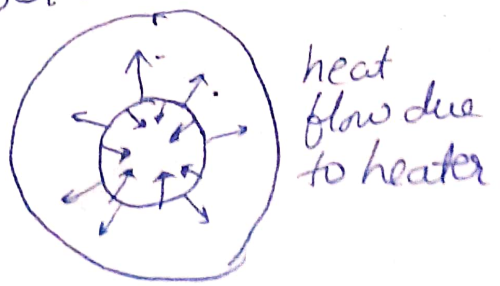
$h = 50$

$q_0' = ?$ (heater output per unit length)



$$q_0'' = \frac{5-(-15)}{\frac{1}{hA}} = \frac{20}{\frac{1}{\pi D L h}}$$

It does not split here. All heat goes through cylinder & convection.
As the heater is connected to rod, so some heat goes inside and some outside. So the rod gets hotter & hotter. But that is not possible as problem states steady state.



(Noth

No heat goes into middle of rod; because it can't get out, it can't escape & that wouldn't be a steady state problem.
(Tricky Problem).

$$q_0' = \frac{q_0}{L} = \frac{1}{k}\left[\frac{20}{\frac{1}{\pi D k h}}\right]$$

$$= \frac{20}{\frac{1}{\pi D h}}$$

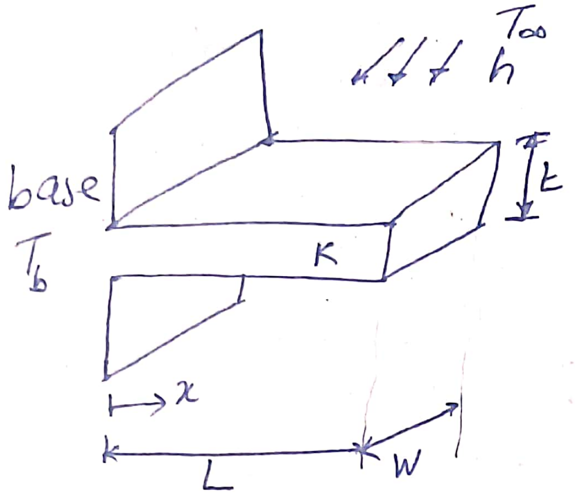
$$q_0' = 20 \times \pi D h$$

Now, we can get heater temperature, as we are worried about it getting hot and failing.

$$q_0' = \frac{T_{heater} - (-15)}{\frac{\ln(r_2/r_1)}{\pi D k} + \frac{1}{\pi D h}}$$

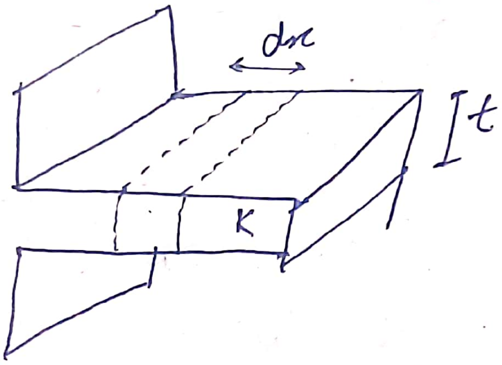
gives value of T_{heater}.

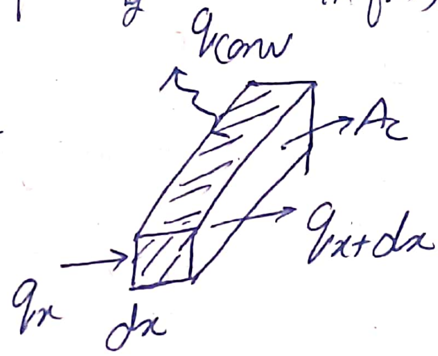
Extended Surfaces (fins).



Rectangular Fin

Assumptions:-
- steady state
- 1D
- K constant
- h is uniform on surface
- $\dot{q}_g = 0$ (no heat generate in fin)





$$\dot{E}_{in} - \dot{E}_{out} = 0$$ (No heat Generation) (steady state)

$$q_x - q_{x+dx} - q_{conv} = 0$$

$$q_{x+dx} = q_x + \frac{d}{dx}(q_x)\,dx$$

$$q_x = -kA_c\frac{dT}{dx}$$

$$q_{conv} = hA(T - T_\infty)$$
$$= hP\,dx\,(T - T_\infty)$$

Area for convection = $P\,dx$

P = Perimeter

Using enagy balance

$$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}(T - T_\infty) = 0$$

let $\theta = T - T_\infty$ | $\frac{d\theta}{dx} = \frac{dT}{dx}$

let $m = \sqrt{\frac{hP}{KA_c}}$

gives $$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Sol^n:- $\theta(x) = c_1 e^{mx} + c_2 e^{-mx}$.

We need to find c_1 & c_2.

BC #1 at $x = 0$, $T(x=0) = \theta_b$

$\theta_b = T_b - T_\infty$

BC #2 One of the following

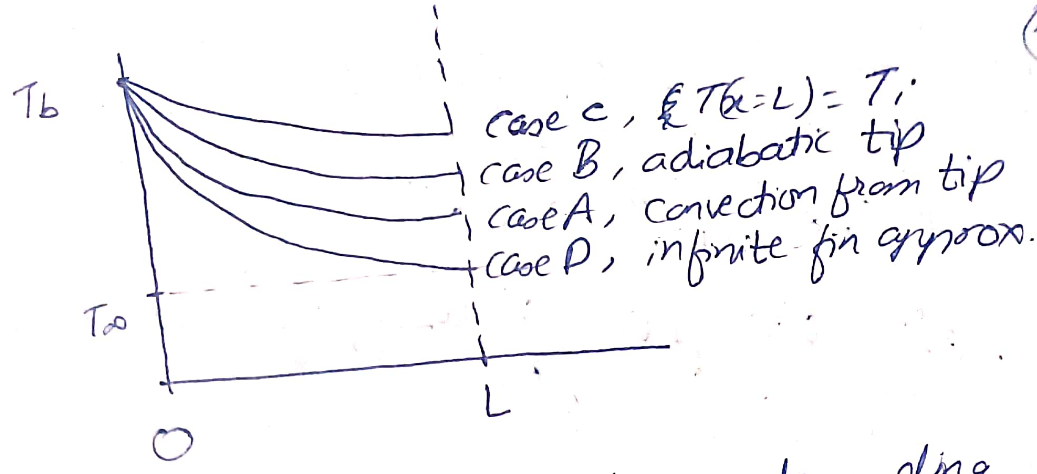
Case A : Convection from fin tip $(x = L)$

Case B : adiabatic fin tip

case C : Given temp at fin tip.

Case D : Infinite long fin $(T \rightarrow T_\infty)$
(fin tip approaches T_∞)
(only if $mL \geq 2.65$)

other case can be radiation from fin tip.



A, B, C can be top or bottom depending upon temperature.

Only for case D it is totally correct.

We solve for c_1 & c_2.

For uniform ~~crossect~~ section Area.
(Table)

Case	Tip Condition.	θ/θ_b	q_b
A	Convection		
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Given temp at fin		
D	Infinite fin approx.	e^{-mx}	M

$m = \sqrt{\frac{hP}{kA_c}}$ Base heat ~~temp.~~ transfer $(M) : \sqrt{hPkA_c}\,\theta_b$

For non-filled columns values are large

We get θ/θ_b by solving

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Then we get q. using fouriers law.

To get q_f

$$q_f = -KA\frac{dT}{dx}\Big|_{x=0}$$

All the heat lost to find comes base.

So, we get of q_f.

Or q_f = leaving the fin by convection.

First method is easier.

We can't use our last table for fins like

The cross section area changes.

Fin Efficiency

$$\eta_f = \frac{q_f}{q_{max}} = \frac{\text{actual heat loss by fin}}{\text{maximum heat that could be lost by fin.}}$$

$$\boxed{\eta_f = \frac{q_f}{hA_f\theta_b}}$$

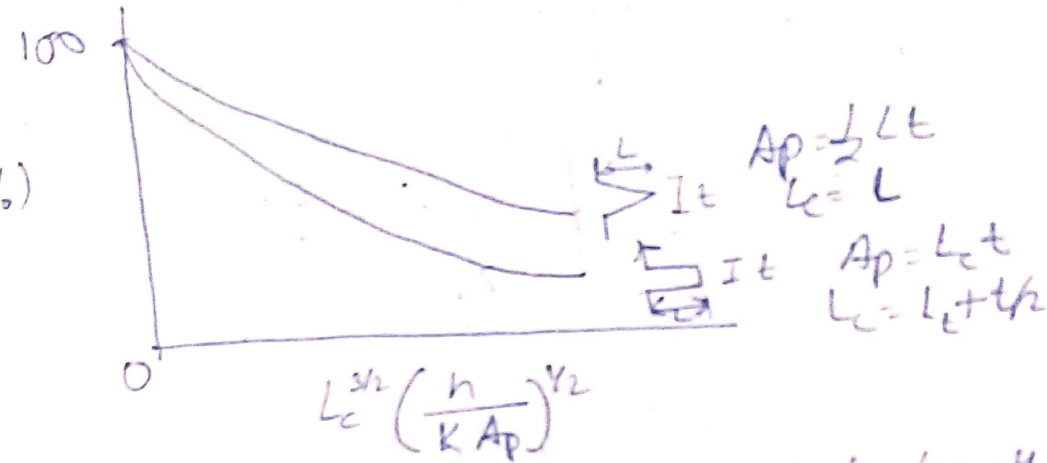
A_f = fin area.

$\theta_b \rightarrow (T_b - T_\infty)$

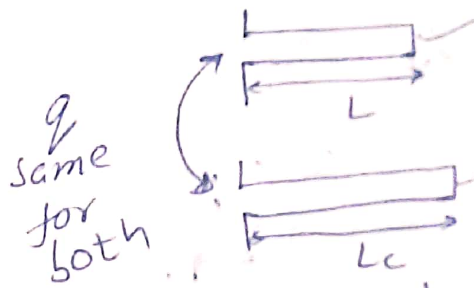
Highest heat transfer: $h A_f \theta_b$

$(T_b - T_\infty)$

For all 6 cases, ~~fin tea~~ the temp doesn't go down much at all.



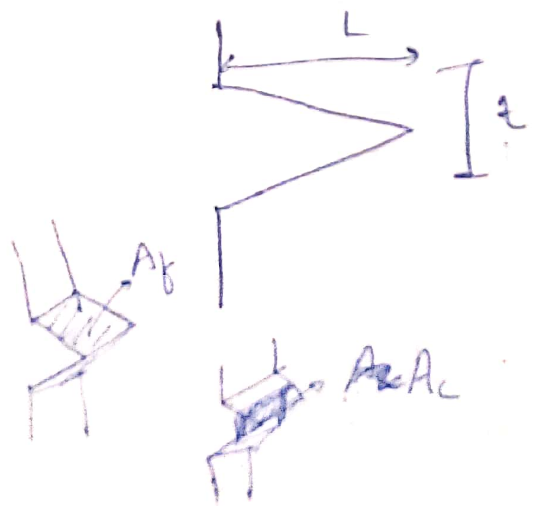
A_p = Profile Area (looking from sideways), L_c = corrected fin length.



$L_c > L$

~~tee~~

For triangular fin



$A_p = \frac{1}{2} \times L \times t$ (side profile area)

$L_c = L$, because there is no area at end of fin.

We have 3 areas →

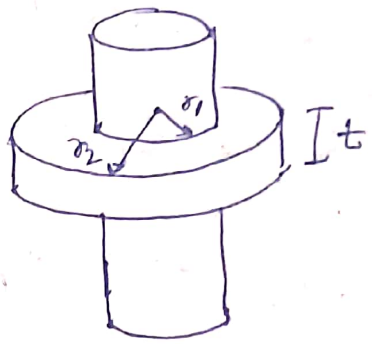
A_p - Profile area. side view of fin.

A_c - Cross section area through which heat is conducted

A_f - Fin area (fin area that touches the fluid)

(if fin ~~is adi last~~ tip is adiabatic, it does not touch the fluid, so not included in A_f)

Circumferential fins.

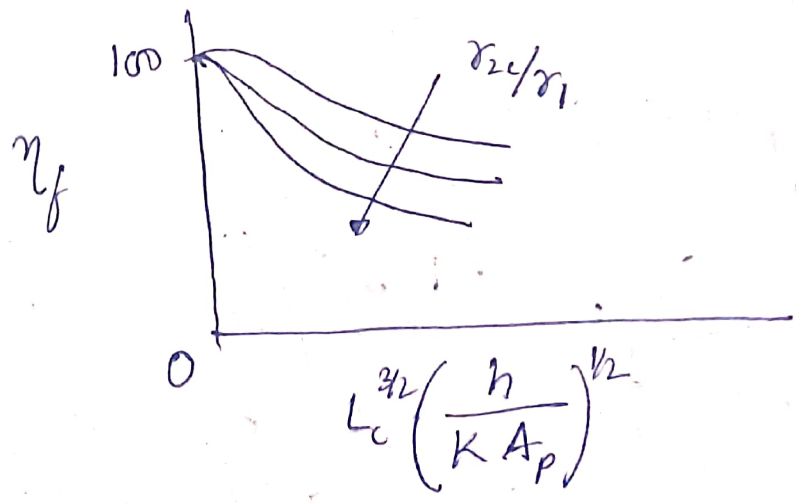


$r_{2c} = r_2 + t/2$

$L_c = L + t/2$

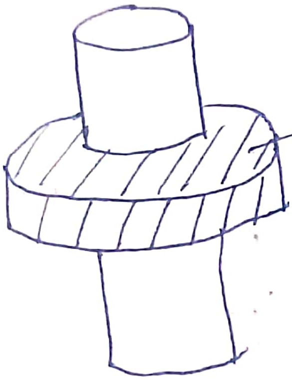
$A_p = L_c t$

We have plot of η vs $L_c^{3/2}\left(\frac{h}{KA_p}\right)^{1/2}$



We find η & put into

$q_f = \eta_f \times h A_f \theta_b$

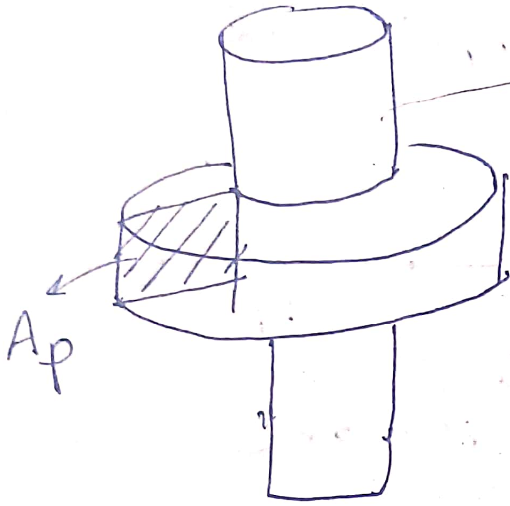


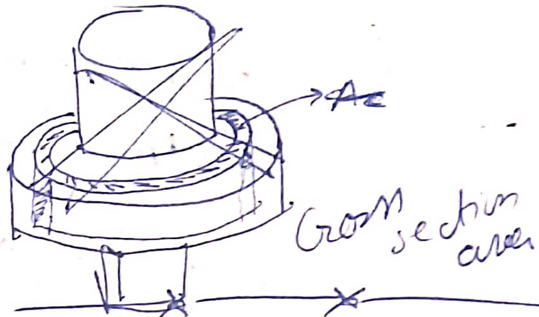
A_f (Area that touches the fluid)

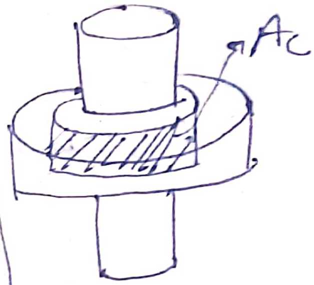
$$A_f = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$

$$A_p = t(r_2 - r_1)$$

Profile area.



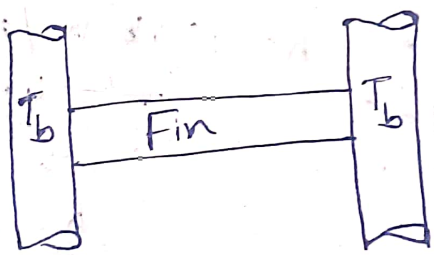


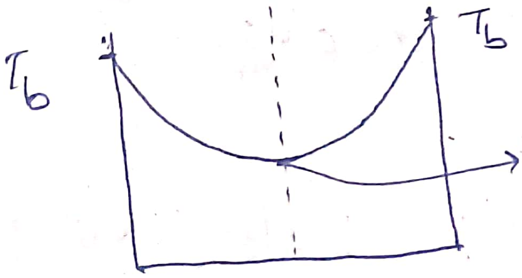


$$A_c = 2\pi r \times t$$

Lec-9

Consider the case

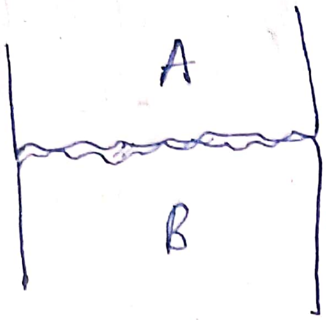




$$\frac{dT}{dx} = 0$$

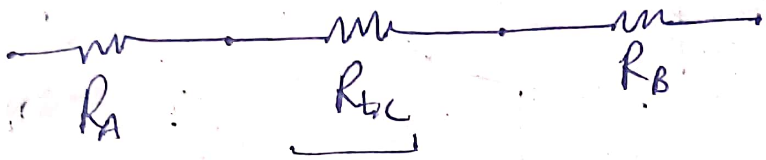
This is case B in table i.e. adiabatic fin tip. We model half of the fin with adiabatic fin tip BC.

Contact Resistance



Air spaced b/w them.

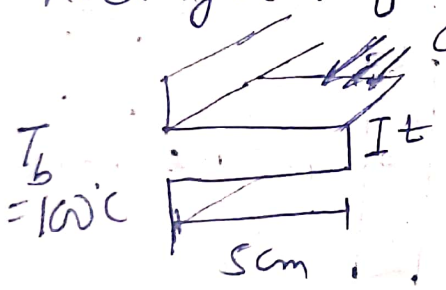
When there is air trapped, the resistance increases.



Constant resistance.

Example

Rectangular fin



air at 20°C

$h = 100 \; W/m^2K$

$w = 20 \; cm$

$t = 0.5 \; cm$

$K = 180 \; W/mK$

fin is 2024 al alloy

Find q_f.

We get K by looking at table of Al Alloy at temp $\frac{100+20}{2} = 60°C$ & value of K taken as 180 W/mK

(solution from Type 1)

It is a fin with uniform cross section.

It is convecting heat away.

we can use case D. if

$mL_c \geq 2.65$.

if it is less than 2.65, we use case A.

$m = \sqrt{hpKA_c}$ $m = \sqrt{\frac{hP}{KA_c}}$

$$m = \sqrt{\frac{100 \times 2 \times (0.2 + 0.05)}{180 \times (0.2 \times 0.05)}}$$

$$= \sqrt{\frac{100 \times 2 \times 0.205}{180 \times 0.2 \times 0.05}} = 15.09$$

$$mL = (15.09) \times 0.05 = 0.755 \leq 2.65.$$

$mL \leq 2.65$, so we use case A.

Case A will always be correct. Case D will only be correct if $mL \geq 2.65$.

(For case see table 6 pages back before fin efficiency)

Case A:

$$q_f = M \left[\frac{\sinh mL + \frac{h}{mK} \cosh mL}{\cosh mL + \frac{h}{mK} \sinh mL} \right]$$

$$M = \sqrt{hPKA_c}\,\theta_b$$

$$= \sqrt{100 \times (2 \times 0.0205) \times 180 \times (0.2 \times 0.05)} \times (100 - 20)$$

$$= \cancel{\sqrt{73.8} \times 80} \quad \sqrt{7.38} \times 80$$

$$= \cancel{687.255} \quad 217.32$$

$$q_f = \cancel{687.255}\ 217.32 \times \left(\frac{\sinh 0.755 + 0.037 \cosh 0.755}{\cosh 0.755 + 0.037 \sinh 0.755} \right)$$

$$= \cancel{687.255}\ 217.32 \times \left(\frac{0.8288 + 0.037 \times 1.298}{1.298 + 0.037 \times 0.8288} \right)$$

$$= \cancel{687.255}\ 217.32 \times \left(\frac{0.876826}{1.3286656} \right)$$

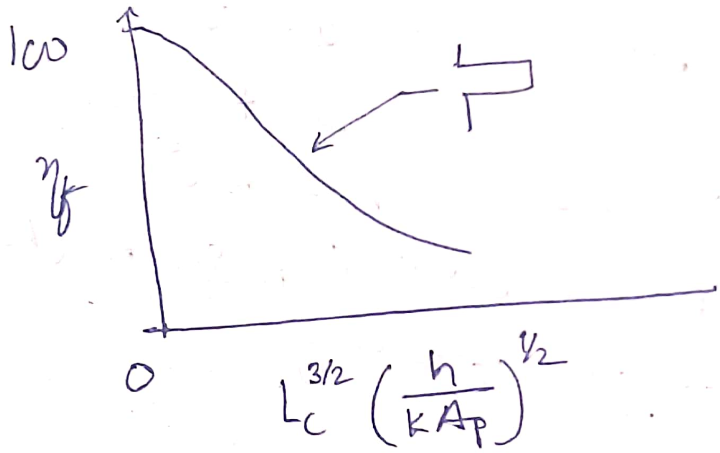
$$= \cancel{687.255}\ 217.32 \times 0.66$$

$$= \cancel{453.5\ W}\ 143.43\ W.$$

It is heat loss by a single fin.

OR, we can use figure for calculations

solution from (type-2)



$$L_c = L + t/2 = 0.0525$$

$$A_p = L_c \times t = 0.0525 \times 0.0035$$

$$= 0.0002625$$

$\cancel{Ke =}$

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.0525)^{3/2} \times \left(\frac{100}{180 \times 0.0002625}\right)^{1/2}$$

$$= 0.5533$$

From graph: $\eta_f \cong 0.83$

So, $q = \cancel{\sqrt{\ }} \eta_f \times h \times A_f (T_b - T_\infty)$

$= 0.83 \times 100 \times A_f \times (80)$

$A_f = \cancel{2 \times (0.0525) \times 0.005 + 2 \times 0.05 \times 0.20}$

$\cancel{0.0205}$

$=$

$q = \cancel{136.12}$

$A_f = 2 \times (\underset{L_c}{0.0525} \times \underset{t}{0.005} + 2 \times \underset{L_c}{0.0525} \times \underset{w}{0.20}$

$= 0.021525$

$q = 0.83 \times 100 \times 0.021525 \times 80$

$= 142.926 \text{ W}.$

(We are using L_c in graph. In using L_c we are assuming no heat loss at fin tip.)

So, we get same answer using table or graph.

<u>OR</u> we can use another table

(solution from type 3) $\eta_f = \dfrac{\tanh mL_c}{mL_c}$

$m = 15.09 \quad mL_c = 15.09 \times 0.0525$

$= 0.792225$

$\eta_f = \dfrac{\tanh 0.792225}{0.792225} = 0.8326$

So, we have same effuency that we get same answer from previous two methods.

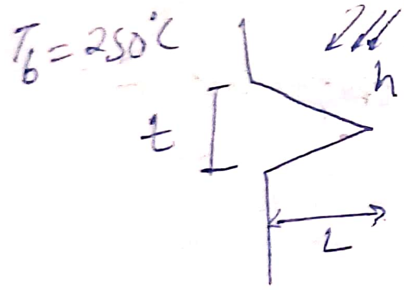
So, $q = \eta \times h A_f (T_b - T_\infty)$

$\simeq 143$ W.

Now, if we want to find temp at fin tip, we can use table (go 9 pages back)

We can use type of condition & use θ/θ_b to get temp at a particular location.

Triangular fin



air $T_\infty = 30°C$ $t = \cancel{2cm}$ 2mm

$h = 40$ W/m²K $L = 6$ mm

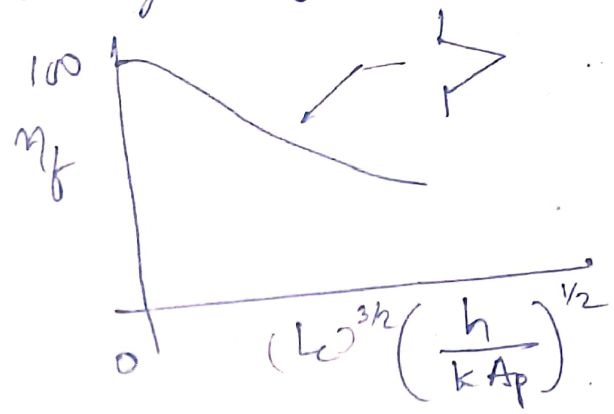
Fin made of Al.

Find K, using table.

at $\frac{250+30}{2} = 140°C$, value of k from table $= 240$.

Find q' (W/m).

For triangular fin



$L_c = L = 0.006$

$A_c = \frac{L_c \times t}{2} = \frac{0.006 \times 0.002}{2}$

$= 6 \times 10^{-6}$

$\Rightarrow (L_c)^{3/2}\left(\frac{40}{240 \times 6 \times 10^{-6}}\right)^{1/2} = (0.006)^{3/2} \times 166.66$

$= 0.0774$

From graph, $\eta_f \simeq 0.98$.

$q_f = \eta_f \times h A_f (T_b - T_\infty)$

$= 0.98 \times 40 \times A_f \times (250 - 30)$

$A_f = \left(\cancel{b \times} w \sqrt{t_{1/2}^2 + L^2}\right) \times 2$

$= \cancel{b} w \sqrt{(0.001)^2 + (0.006)^2} \times 2$

$= w \times 0.0122$

$q_f/w = 0.98 \times 40 \times 0.0122 \times 220$

$q' = 105.213$ W/m

OR we can use another table.

$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$

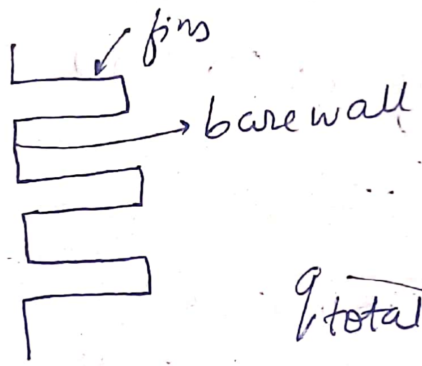
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$ Bessel fn.

& we get $\eta_f = 0.96$.

Similarly we get q_f.
And answers are very similar.

For triangular fin the fin tip goes down to zero area. So, it is adiabatic at end. So, we use L_c same as L for ~~pin f~~ triangular fin.
Similar is case for parabolic fin

Finned Surfaces



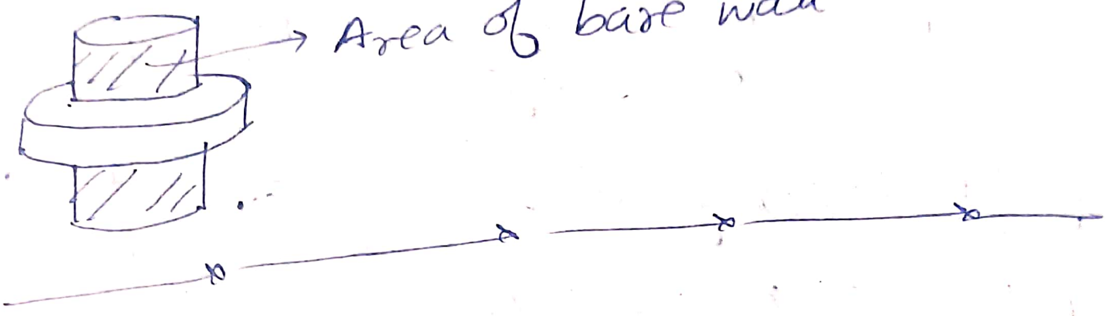
$q_{total} = q_{fins} + q_{bare\ wall\ area}$

$$\cancel{q_{total} = h\left[N \eta_f A_f + A_t - A_f\right.}$$

$$q_{total} = h\left[\underbrace{N}_{\#\text{ of fins}} \eta_f A_f + (\underbrace{A_t}_{\text{total area}} - N A_f)\right]\theta_b$$

$$= hA_t\left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\theta_b$$

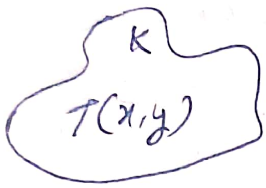
($A_t - NA_f$ = Area of bare wall)



Lec 10 :

Two Dimensional Conduction

Assume SS, no generation, constant property



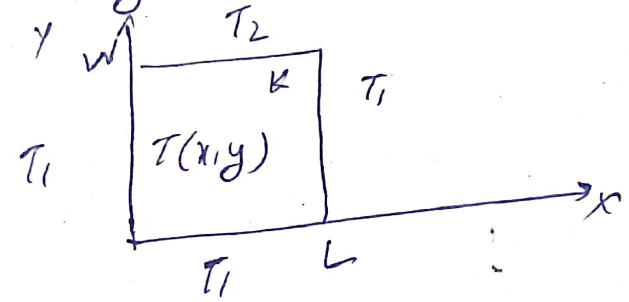
governing PDE is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

This can be solved by

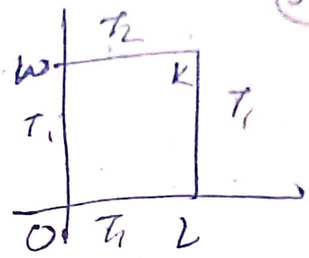
1. Analytical method
2. Numerical method
3. Graphical techniques

Analytical method



we have 4 BCs & need 4 BCs

DE : $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$



BC #1 : $T(x, y=0) = T_1$

BC #2 : $T(x, y=w) = T_2$

BC #3 : $T(x=0, y) = T_1$

BC #4 : $T(x=L, y) = T_1$

We solve DE with those BCs.

Sol^n (by separation of variables is

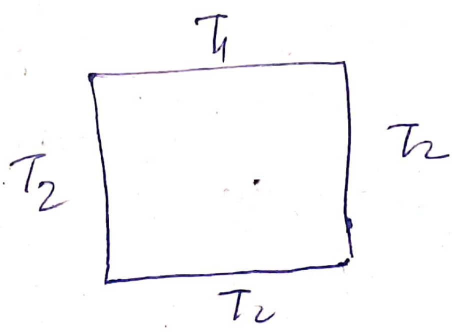
$$\frac{T - T_1}{T_2 - T_1} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}+1}{n} \frac{\sin\frac{n\pi x}{L} \sin\left(\frac{n\pi y}{L}\right)}{\sin\left(\frac{n\pi w}{L}\right)}$$

This is a sol^n for a rectangular plate.

We will use graphical technique.

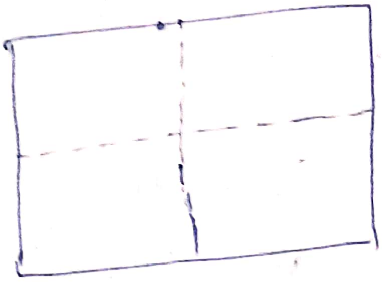
We will create a flux plot of adiabats and isotherms.

* Construct flux pplot for.

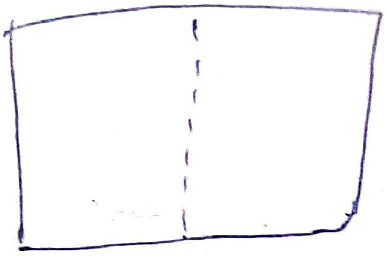


First we identify lines of symmetry (Geometrical & thermal)

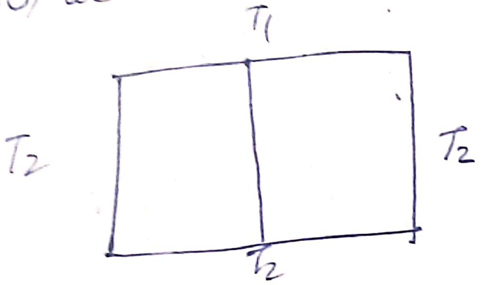
Geometrical Symmetry



Thermal symmetry



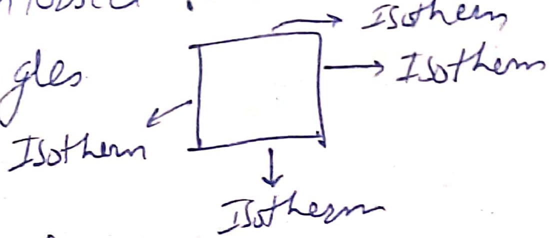
~~Th~~ So, we keep line that satisfy both

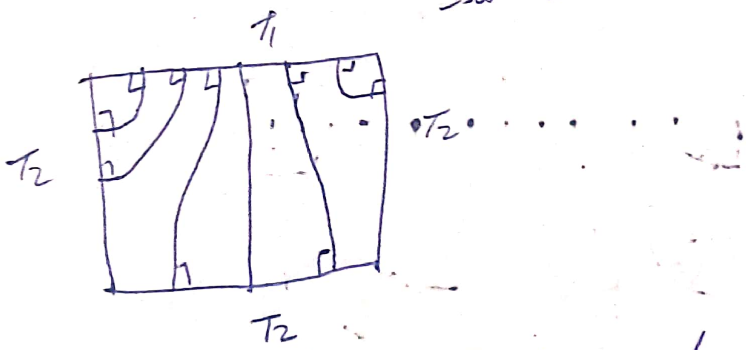


*** (Adiabat is a heat flow line)

* These line of symmetry is adiabats. Adiabat is a heat flow line. ~~Heat~~ In this case $T_1 > T_2$ heat goes down to the bottom.

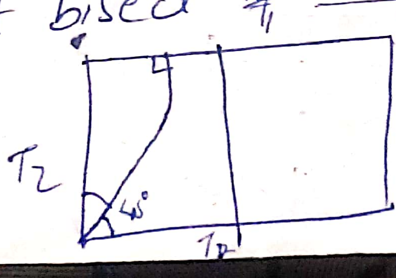
* Adiabats intersect isothermal boundaries at right angles





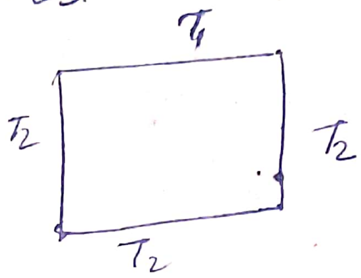
* Adiabat bisect isothermal corners.

(isothermal corners)



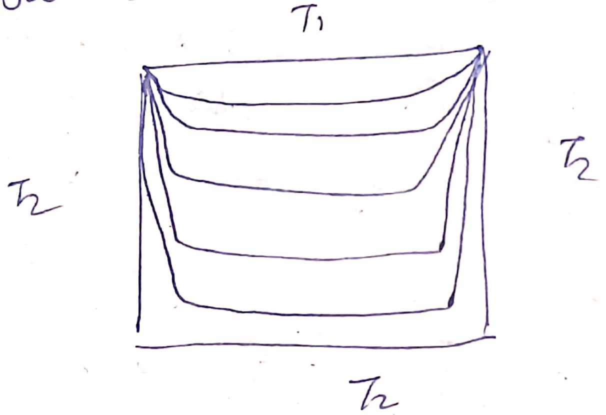
* Now, we will draw constant temperature lines.

We have



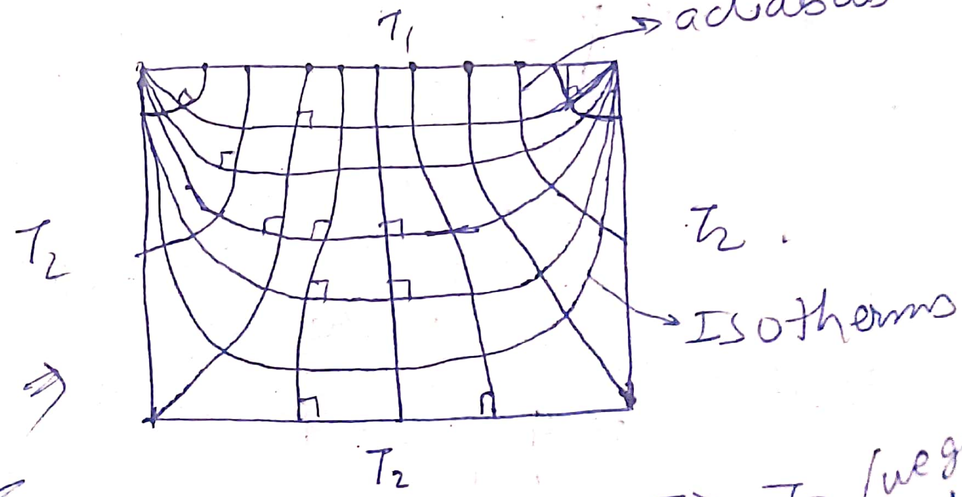
∴ Top surface is T_1 (isothermal line)
Left, bottom & right are at T_2 (isothermal line)

⇒ So, our constant temp line looks as :



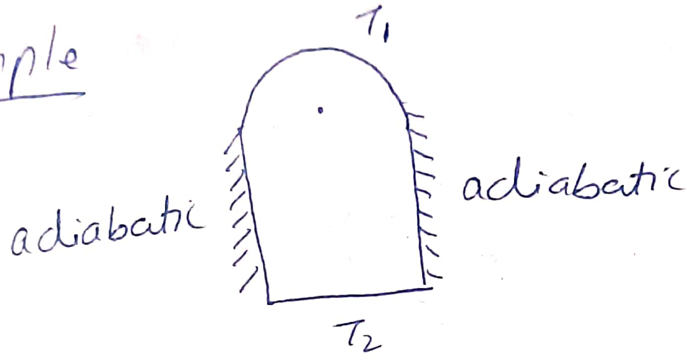
* Inside the solid boundary adiabats & isotherm cross at right angles

Flux plot ⇒

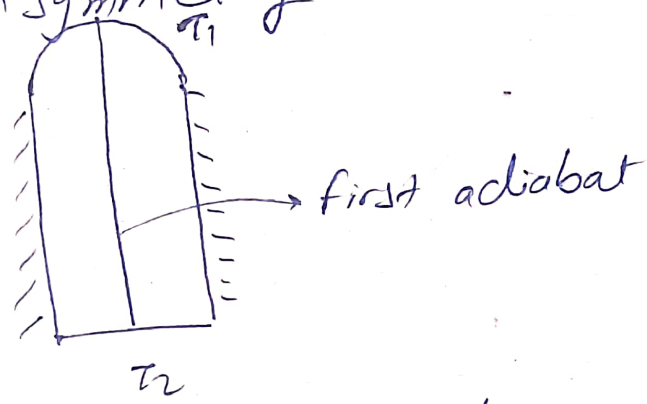


assume $T_1 > T_2$ or $T_2 > T_1$ (we get same plot)

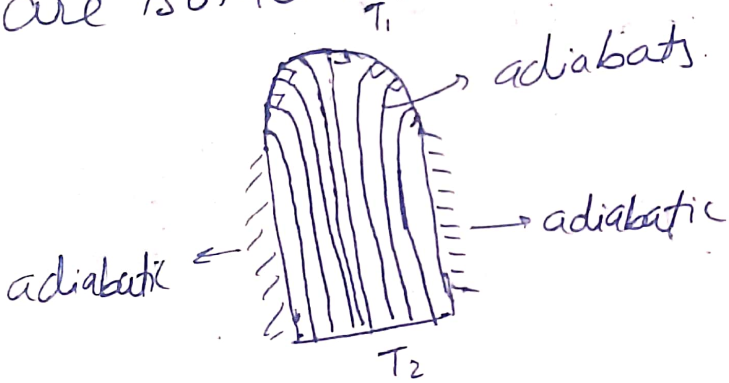
Example



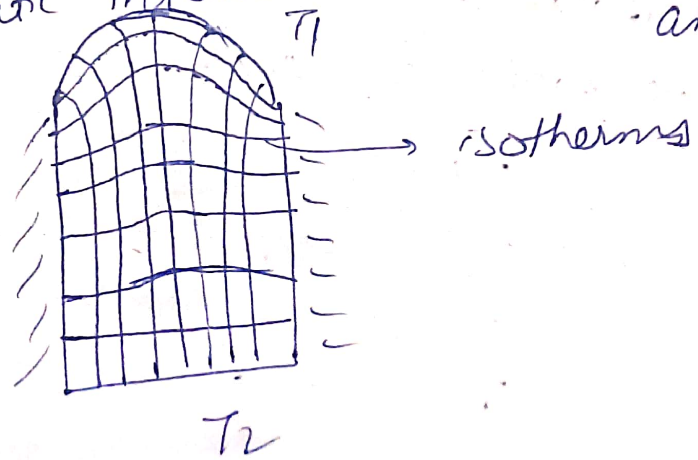
(1) Look for symmetry



(2) Adiabats intersect isotherm at right angle, T_1 & T_2 surface are isotherms.

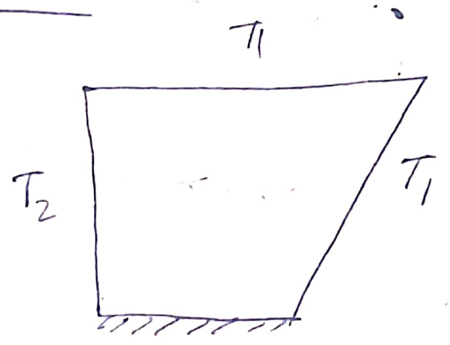


(3) Now we can draw isotherm pretending $T_1 = 100$, $T_2 = 0$. $T_1 > T_2$
(Adiabatic intersect isotherm at right angles)

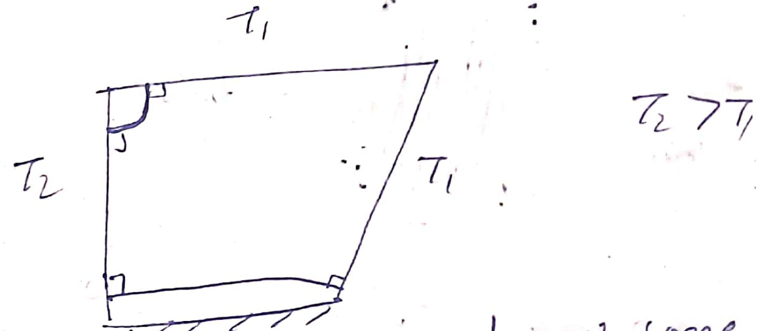


We get that adiabatic & isotherm cross each other at 90° by the laplace eq^n $\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0\right)$

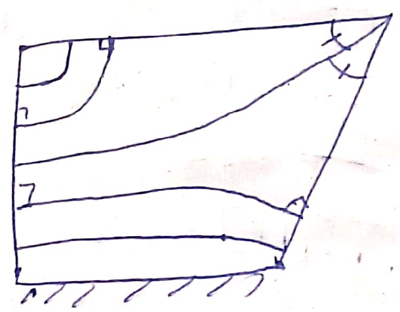
Example



Draw the flux plot.
There are not lines of symmetry
Our initial ~~tr~~ adiabats look as

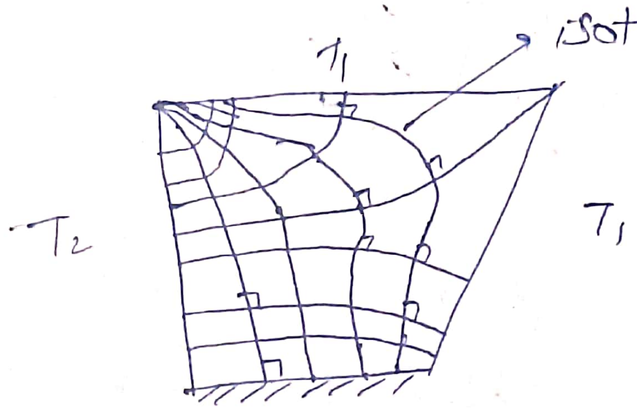


Similarly we can draw some more.
At isothermal corner, adiabat bisects the isothermal corner



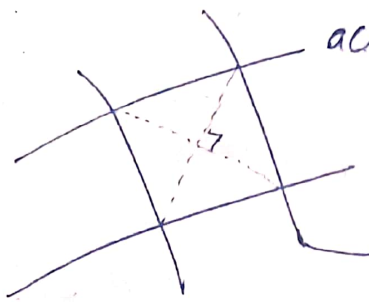
Isotherms come out in a fan patter

So, in our plot.



Flux Plot.

To get an estimate for q we must construct curvilinear squares



We have got a good curvilinear square when diagonals intersect at right angles.

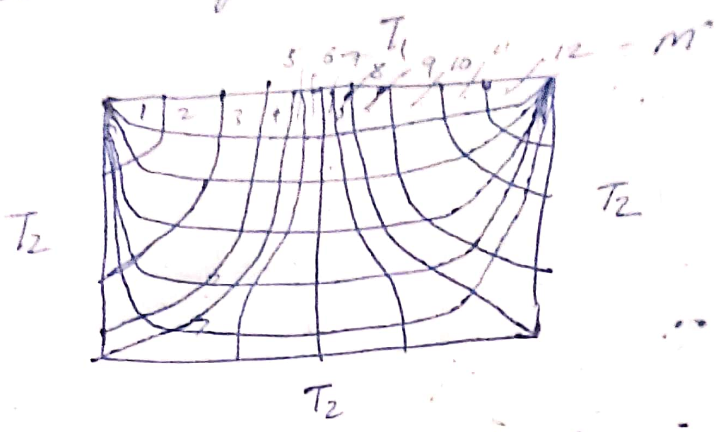
Then $q \cong \frac{ML}{N} K(T_2 - T_1)$

M = no. of heat flow lanes

N = no. of equal temp increments

L = plate thickness.

In our rectangular heat flux plot

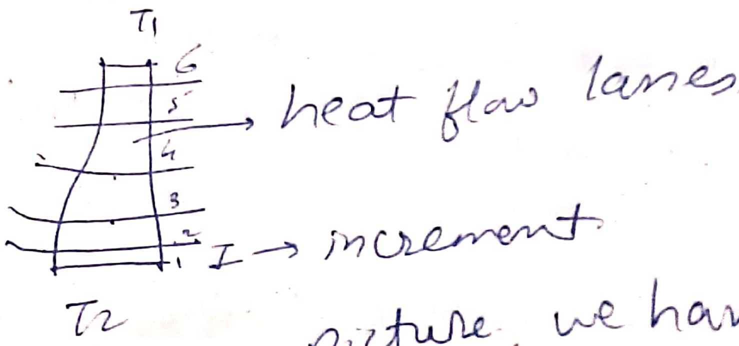


We need to draw such that adiabat & isotherms are 90° & the curvilinear squares are good i.e. have diagonals at 90°. If not we draw again

Now, we use q eqn

$$q \simeq \frac{ML}{N} K(T_2 - T_1)$$

M = no. of heat flow lanes.



So, in our picture we have

M = 12 (12 no. of heat flow lanes)

N = no. of increments.

We have 6 increments in figure (main figure above & also below has 6)

$L = 2\,cm$ (assuming)

$T_1 = 60^\circ C$

$T_2 = 0^\circ C$

$K = 200\ W/mK$

assumed.

(originally provided in question)

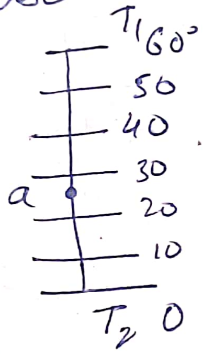
So, we get

$$q = \frac{12 \times 0.02}{6} \times 200\,(60-0)$$

$$= 12 \times 2 \times 2 \times 10 = 480\,W$$

So, we can get estimate of q.

There is also a way to get estimate of T.



Temp at point a is around 25°C.

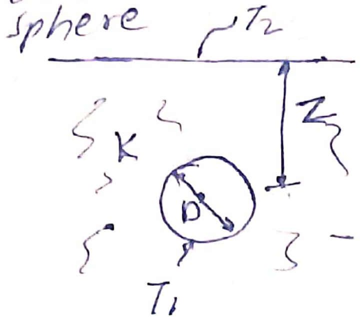
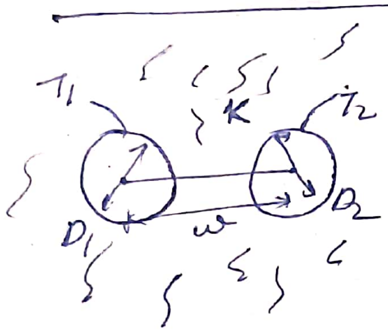
So, we can get estimate of temperature.

We do this sketch, so that we can avoid solving of PDEs. & get a estimate.

By sketching, we can also see if the numerical method gives out a correct number or not (just verify).

Conduction shape factor, S

$$\dot{q} = SK(T_1 - T_2)$$

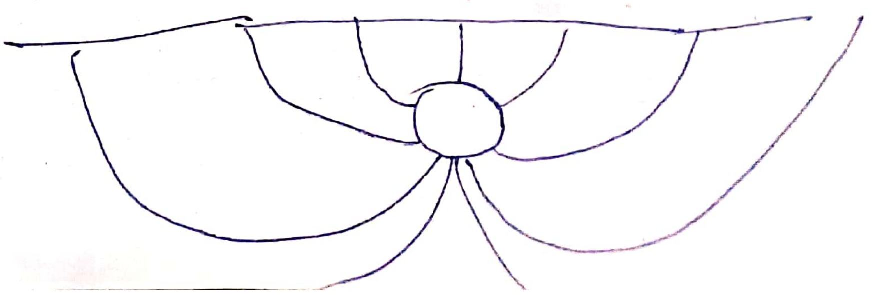
System	Shape Factor, S	Restrictions
Buried sphere 	$\dfrac{2\pi D}{1 - \frac{D}{4Z}}$	$Z > D/2$
 Two separated cylinders	$\dfrac{2\pi L}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$	$L >> D_1, D_2$ $L >> w$ (L = Length) w = space b/w two cylinder

Two cylinder are buried under ground, infinitely under grand & infinitely spaced from all 4 side (Top, bottom, left, right)

For buried sphere

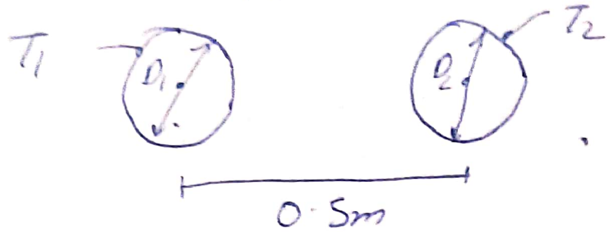
flux plot

adiabats



Ex 2 parallel pipelines buried in soil

K(soil)



$K_{soil} = 0.5\ W/mK$

$D_1 = 100\ mm$

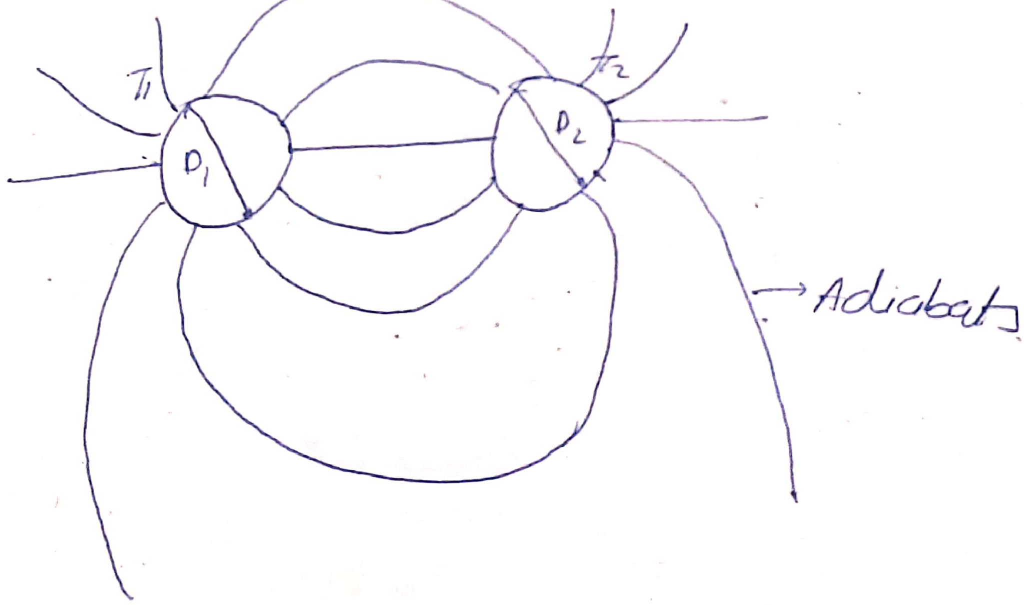
$D_2 = 75\ mm$ find q'

$T_1 = 175°C$

$T_2 = 5°C$

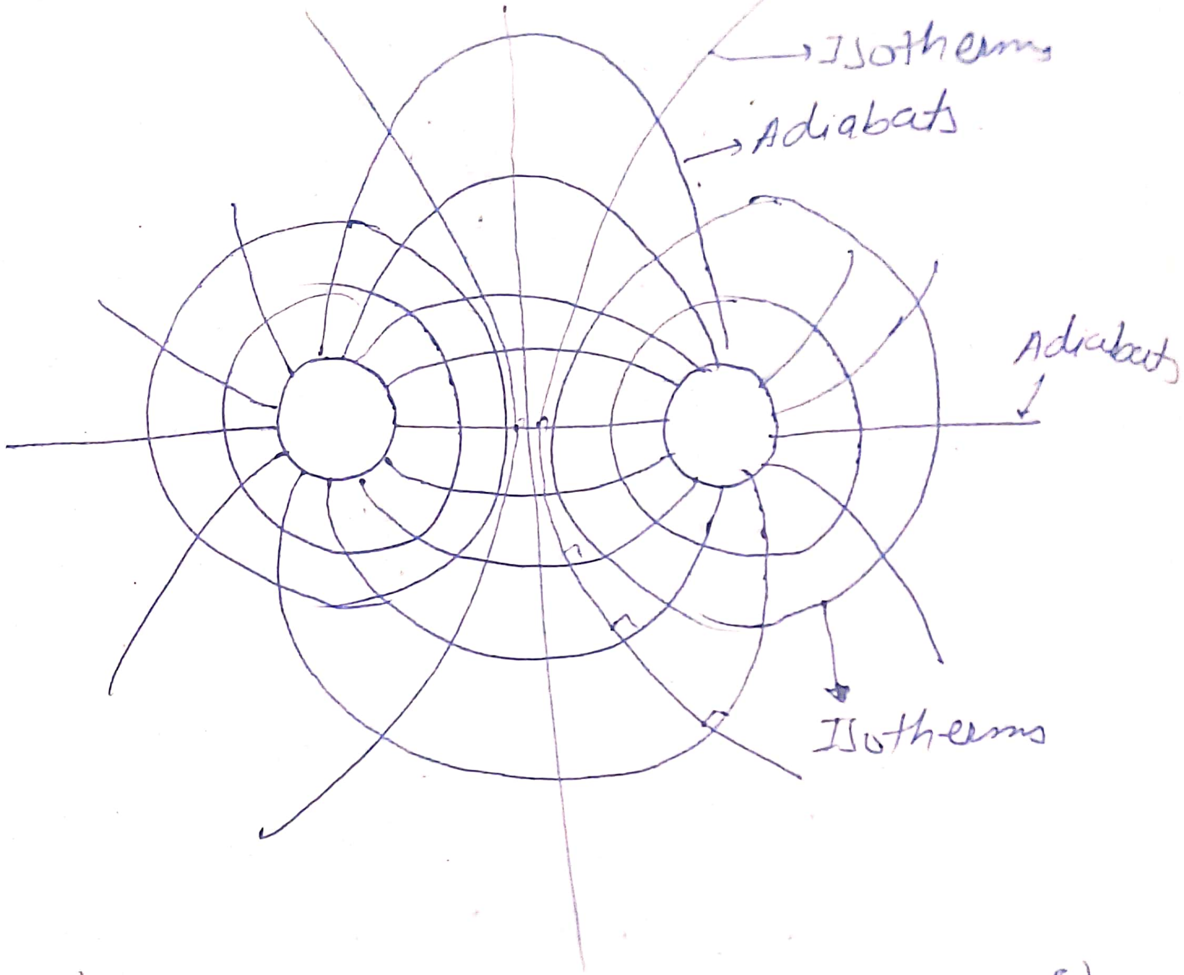
We don't know how long pipe is so we find q'.

∴ We will first draw flux plot.
We have horizontal line of symmetry



(Adiabatic wall - Its a wall where no heat crosses)

Our isotherm will be 90° to adiabats



We need to find q' (shape factor - S)

$$q = SK(T_1 - T_2)$$

$$q' = \frac{SK}{L}(T_1 - T_2)$$

$$= \frac{\cancel{K}}{\cancel{K}}\left(\frac{2\pi K}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}\right)(175-5)$$

$$q' = \frac{2\pi K \times 170}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$

$= 110\ W/m$

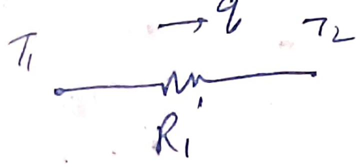
Recall that

$$q = \frac{T_1 - T_2}{R}$$

So, $R = \frac{1}{SK}$.

So, for this case $R' = \frac{1}{S'K}$

(we use R' ~~i.e.~~ ~~R~~ which is $R \times L$.)



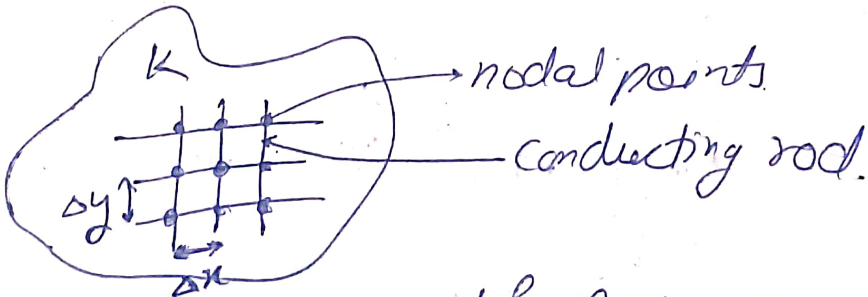
$$q' = \frac{T_1 - T_2}{R_1'}$$

Here $R' = 1.55$.

This is useful where we also have convection outside of the cylinders, ~~or~~ or conduction resistance if any present.

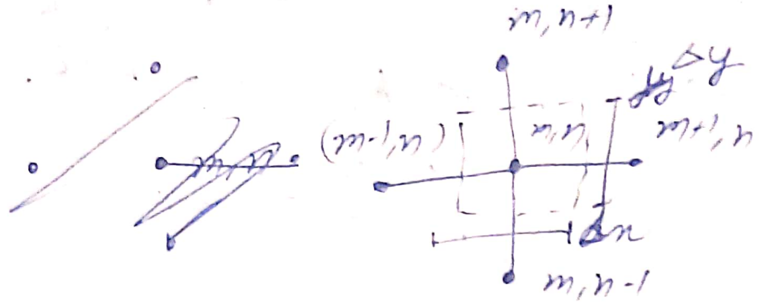
Numerical methods

homogenous body



We ~~ass~~draw the grid & assume mass at nodal points.
The nodes have a temp. & exchange energy with neighbours through heat conducting rod.

We do have approximations but heat is conducted through heat conducting rods.



We place node m,n in a centerline. (dash line)

We will do energy balance on center node.

$$\sum q_{\substack{into \\ (m,n)}} = 0.$$

(We know all can't come in, some will come in & some will go out)

$$q_{(m-1,n)\to(m,n)} + q_{(m+1,n)\to(m,n)} + q_{(m,n-1)\to(m,n)} + q_{(m,n+1)\to(m,n)} = 0$$

$$\frac{K(\Delta y)(1)}{\Delta x}(T_{m-1,n} - T_{m,n}) + \frac{K(\Delta y)(1)}{\Delta x}(T_{m+1,n} - T_{m,n}) + \frac{K(\Delta x)(1)}{\Delta y}(T_{m,n-1} - T_{m,n}) + \frac{K(\Delta x)(1)}{\Delta y}(T_{m,n+1} - T_{m,n}) = 0$$

If our grid is square

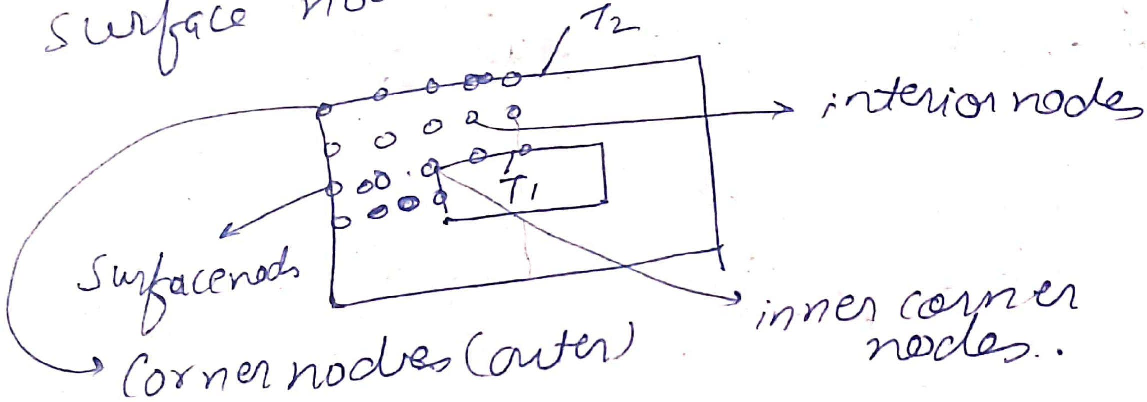
$\Delta x = \Delta y$

Conclusion

$$\boxed{T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = 0}$$

Interior node finite difference eqn.

We have different equations for the type of nodes. like for interior nodes, surface nodes, corner nodes etc.



Then we get eqn's for every type of nodes.

And we solve those eqn's for all the nodes to get the result.

There is also eqn for convection.

Once we have temperature. we can know how ~~to~~ ~~much~~ much heat flows.

Lec-12

Finite Difference eqⁿs

Case 1 — Interior node

with convection:

2 — Internal corner node

3 — Plane wall node.

4 — External corner node

if adiabatic set $q''=0$:

5 — Plane surface with uniform heat flux, q_0''

For an adiabatic surface, set h (in 2,3,4) or q'' (in 5) equal to zero (cases 2, 3, 4 and 5)

(These 5 eqⁿs are used to set a problem & solve)

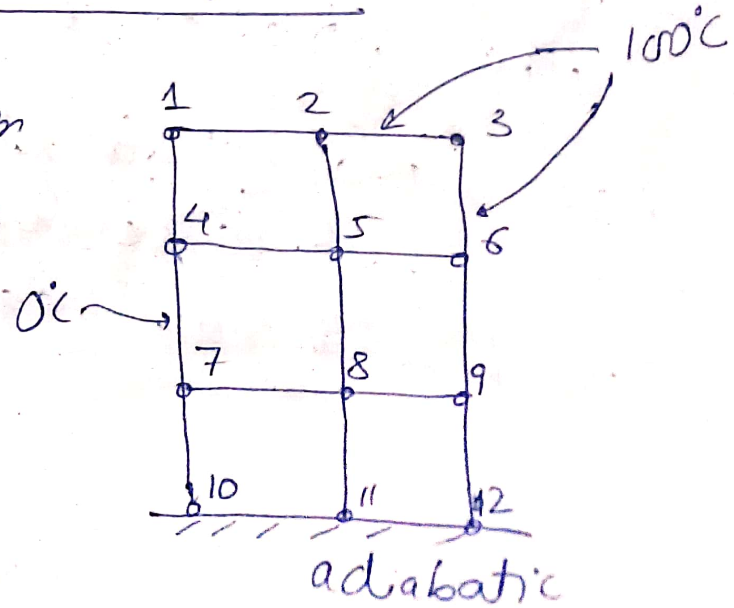
Finite Difference Example

$\Delta x = \Delta y = 0.10\ m$

$K = 25\ W/mK$

Left side = 0°C

Top & right side = 100°C



Give estimate of unknown nodal temp & heat transfer (q).

We know some temperatures

$$T_2 = T_3 = T_6 = T_9 = T_{12} = 100°C$$

$$T_4 = T_7 = T_{10} = 0°C$$

unknown $T_5, T_8, T_{11}, \cancel{T_1}$

For node 1, it is undetermined, as as if we approach from left it is 0 but when we approach in top side, it is 100°C.

So, it is undetermined.

So, it is can't happen in real work.

(It can't happen in real work, we put a thermal insulation such that it does not touch each other)

Node 5 is interior node, so we write

$$T_2 + T_4 + T_6 + T_8 - 4T_5 = 0$$

Node 8 is interior

$$T_5 + T_7 + T_9 + T_{11} - 4T_8 = 0$$

Node 11 (adiabatic boundary) (case 3, h = 0)

$$2T_8 + T_{10} + T_{12} - 4T_{11} = 0$$

Plug in known temperatures

$$T_8 - 4T_5 = -200$$

$$T_5 - 4T_8 + T_{11} = -100$$

$$2T_8 - 4T_{11} = -100$$

(3 eqⁿs, 3 unknowns)

$$T_5 = 63.5°C$$

$$T_8 = 53.8°C$$

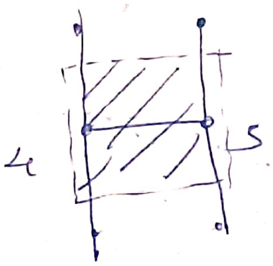
$$T_{11} = 51.9°C$$

(If we want better answer, we have to make grid smaller)

Now, we find out how much heat flow. i.e. get estimate of q.

$$q_{\text{into } 0° \text{face}} = q_{5-4} + q_{8-7} + q_{11-10}$$

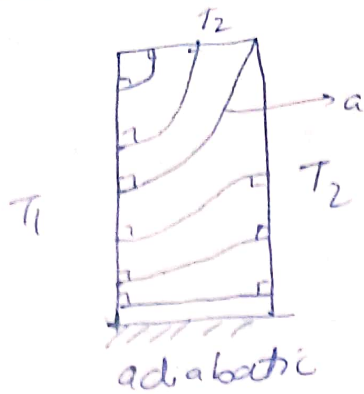
(q_{11-10} → heat can't transfer across the wall, but it can transfer parallel to the wall)
(So, heat will flow from 12 to 10.)

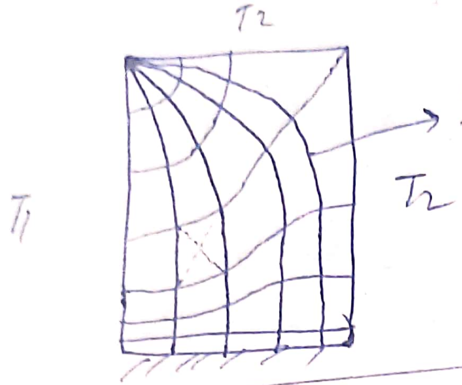


$$q_{5-4} = \frac{k(\Delta y)(1)(T_5 - T_4)}{\Delta x}$$

Heat Flux Plot (for the problem)

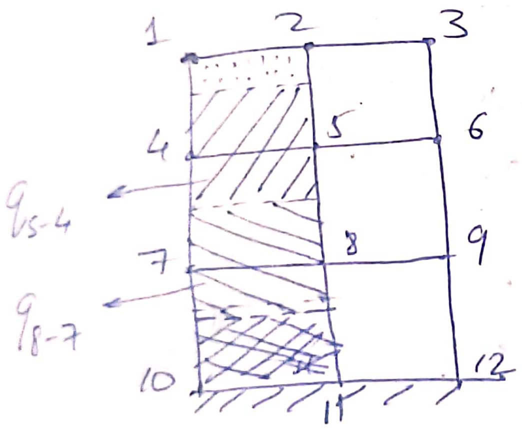
No symmetry.





So we have curvilinear squares.

$$q_{8\to7} = \frac{k(\Delta y)(1)}{\Delta x}(T_8 - T_7)$$



$$q_{11\to10} = \frac{k \times \left(\frac{\Delta y}{2}\right)(1)}{\Delta x}(T_{11} - T_{10})$$

Here $\Delta x = \Delta y$

$$q = k(T_5 - T_4) + k(T_8 - T_7) + \frac{k}{2}(T_{11} - T_{10})$$

$= 360\ W.$

We have ignored the dotted area. But we can if we can increase no. of nodes we can certainly get ~~that~~ the idea of that.

$q_{\text{out of } 100^\circ \text{ face}} =$

$q_{5-2} \; q_{2-5} + q_{6-5} + q_{9-8} + q_{12-11}$

$= 356 W.$

We don't get it equal because our grid size is coarse.

Our approximate answer is

$$\frac{356 + 360}{2} = 380 \text{ W}.$$

Lec-13

Transient Conduction

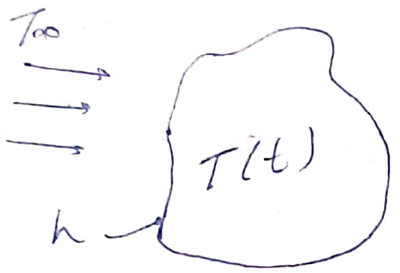
Heat diffusion eqn for constant property, no generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \left(\alpha = \frac{k}{\rho C_p} \right)$$

Solution gives $T(x, y, t)$

~~Th~~ We can use simple model to solve for this.

The simplest possible model is to use the lumped heat capacity model (LHC)



I.C. when $t=0$, $T=T_i$
(We assume temp is only a fn of time)

We don't solve the DE, we write an energy balance on the object

$$\dot{E}_{in}^{\nearrow 0} - \underbrace{\dot{E}_{out}}_{\text{Convection H.T. leaving the body}} - \cancel{\dot{E}_g}^{\nearrow 0} = \underbrace{\dot{E}_{st}}_{\text{change in stored energy of body}}$$

(We are taking energy out of body by blowing over air over it.)

$$\Rightarrow \quad -hA_s(T - T_\infty) = \rho c_p \underset{\text{volume}}{V} \frac{dT}{dt}$$

Let $\theta = T - T_\infty$

$$\Rightarrow \frac{d\theta}{dt} = \frac{dT}{dt}$$

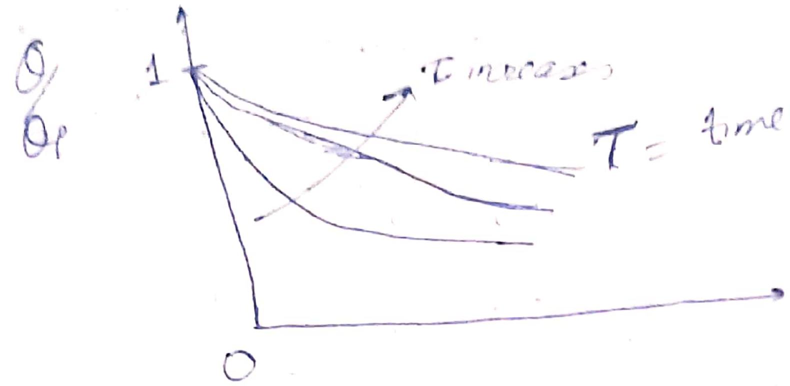
$$-hA_s\theta = \rho c_p V \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{hA_s}{\rho c_p V}\theta$$

IC when $t=0$; $T=T_i$, so $\theta = T_i - T_\infty = \theta_i$

$$\boxed{\frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s}{\rho c_p V} t\right]}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$$



T = time constant (s)

τ = (tau) = Time constant

$$= \frac{\rho c_p V}{h A_s}$$

So,

$$\boxed{\frac{\theta}{\theta_i} = \exp\left[-t/\tau\right]}$$

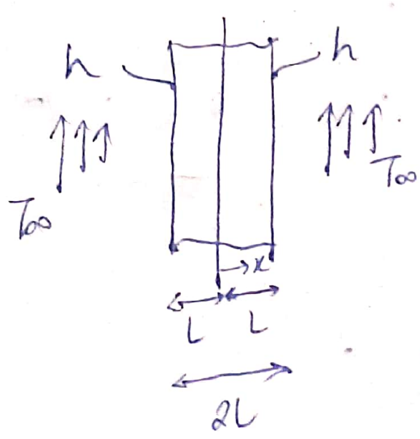
$$\left(\frac{L/KA}{1/hA}\right) \frac{hL}{K}$$

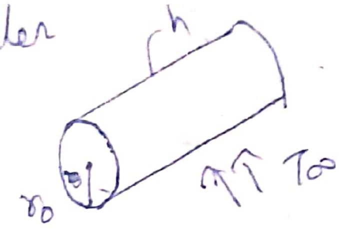
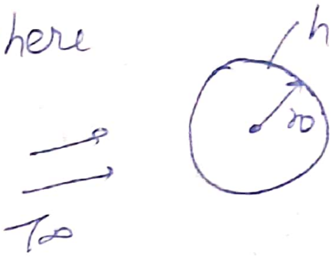
The LHC model can be used when Bi = Biot number $= \frac{h L_c}{K} < 0.10$

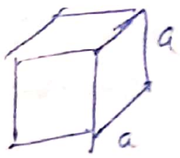
Biot number is ratio of 2 resistances conduction & convection

L_c = characteristic length

$$= \frac{V}{A_s}$$

Geometry	$L_c = V/A_s$
	$L_c = L$ ↑ half thickness

Geometry	$L_c = V/A_s$
Cylinder 	$L_c = r_0/2$
Sphere 	$L_c = r_0/3$. $\left(\dfrac{\frac{4}{3}\pi r_0^3}{4\pi r_0^2}\right) = r_0/3$

So, cuboid 

$$L_c = V/A_s = \frac{a^3}{6a^2} = a/6$$

So, $L_c = V/A_s$.

The energy lost by body (by convection)

$$Q = \int_0^t h A_s \theta \, dt \quad (\theta = \Delta T)$$

(we found temp. distribution, now we find Q)

$$\frac{\theta}{\theta_i} = \exp[-t/\tau]$$

$Q \rightarrow J$ (energy)
$q \rightarrow W$ (power)

(When we talk about energy $\rightarrow Q$)
(When we talk about heat transfer $\rightarrow q$)

this gives

$$\boxed{Q = (\rho c_p V)\,\theta_i\left[1 - \exp(-t/\tau)\right]}$$

Example

Copper sphere ($D = 15cm$)

$T_i = 300°C$, $T_\infty = 30°C$ Air

property at $T_{avg} = 438K$

$\rho = 8933\ kg/m^3$

$c = 397\ J/kg\,K$

$K = 393\ W/mK$

$h_{air} = 25\ W/m^2K$

Find $T(t)$

$\Rightarrow$ check Bi

$$Bi = \frac{hL_c}{K} = \frac{25 \times \cancel{(0.015/3)}}{\cancel{393}}$$

$$= \frac{25 \times (0.0075/3)}{393}$$

$$= 0.00159 < 0.1$$

So, we use

$$\frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s}{\rho c V} t\right]$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-t/\tau\right]$$

$$\tau = \frac{\rho c V}{hA_s} = \frac{\rho c L_c}{h} = \frac{\rho c \frac{r_0}{3}}{h}$$

$$= 3546s$$

$$T = T_\infty + (T_i - T_\infty)\exp(-t/3546)$$

$$T = 30 + 270\exp(-t/3546)$$

After getting T, we can find Q.
Like after 5 minutes how much heat will be lost.
We can also find heat loss by convection eqn i.e. by the energy balance part.

Flux Plot

- Flux plot is a ~~visual~~ visual method used to solve two-dimensional, steady state heat conduction or electrical field problems
- Heat flow lines (adiabats) & isotherm must intersect at 90° angles to form a grid of curvilinear squares.

① Fundamental Boundary Rules:-

a) Isotherms (constant temperature): Heat flow lines always intersect these surfaces at perpendicular (90°) angle.

b) Insulated Boundaries (Adiabatic): No heat crosses these lines. Heat flow lines must run parallel to insulated surface.

c) Lines of symmetry: Treat these lines exactly like insulated boundaries, heat flow lines run parallel to them.

② Construction:

a) Identify Symmetry: Cut geometry into smallest symmetrical section possible to reduce drawing work.

b) Sketch constant increments: Divide total temp. drop (ΔT_{total}) into a whole no. of equal temp steps (n).

c) Start at the bounded channel: Begin sketching in narrowest or most restricted channel where flow direction is predictable

d) Grow the grid outward into wider regions, maintaining 90° intersections and square proportions.

Transient Heat Conduction

Spatial Effects

(LHC model can be used if $Bi < 0.10$)

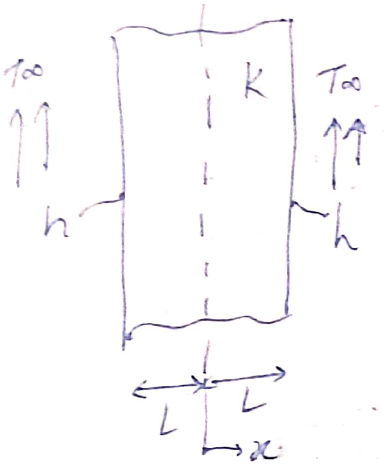
If $Bi > 0.10$

$T(x, t)$ or $T(r, t)$

Temperature depends on x & t.

PDE is (for the plane wall)

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



with correct IC & BC

sol^n is an infinite series.

We will look at 1 term approximation.

Approx sol^n is $(T(x,t))$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{\theta}{\theta_i}$$

$$\theta^* = \frac{\theta}{\theta_i} = C_1 \exp(-\xi_1^2 Fo) \cos(\xi_1 x^*)$$

zeta.

Table 5.1

Bi	Plane wall ξ_1	C_1	Infinite cylinder ξ_1	C_1	Sphere ξ_1	C_1

for wall $Bi = \frac{hL}{k}$ $\left(\frac{hL_c}{k}\right)$

for cyl $Bi = \frac{hr_o}{k}$

for sphere $Bi = \frac{hr_o}{k}$

(in table values are given in form of hr_o/k) for sphere $= \frac{hr_o}{3k}$

and F_o = Fourier number

$$= \frac{\alpha t}{L^2}$$

and ~~$x^* = \frac{x}{L}$~~ $x^* = \frac{x}{L}$

(Eqn is written in dimensionless quantities)

$$\frac{\theta}{\theta_i} = C_1 \exp(-\xi_1^2 F_o) \cos(\xi_1 x^*)$$

For center temperature, (i.e. $x = 0$, $x^* = \frac{x}{L} = 0$)

$$\cos(\xi_1 x^*) = \cos(0) = 1$$

$$\theta_o^* = \frac{\theta}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_o).$$

So, we get center temp. T_o.

For Q

$$\frac{Q}{Q_o} = 1 - \frac{\sin \xi_1}{\xi_1} \theta_o^*$$

where $Q_o = \rho C_p V (T_i - T_\infty)$

Q_o is how much energy is stored initially in the object w.r.t the surrounding fluid temperature T_∞.

For Infinite cylinder (infinitely long)

~~for $T(r,t)$~~

The governing eqn for transient heat conduction in a long cylinder is

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R}\frac{\partial \theta}{\partial R} = \frac{\partial \theta}{\partial F_o}$$

$$\theta = \frac{T(r,t) - T_\infty}{T_i - T_\infty} \qquad R = \frac{r}{r_o} \qquad F_o = \frac{\alpha t}{r_o^2}$$

For $F_o > 0.2$; infinite series soln simplifies to a precise one-term approximation:

$$\theta_{cylinder} = C_1 \exp(-\xi_1^2 F_o) \, J_o(\xi_1 \cdot R)$$

J_o is zeroth order Bessel fn of first kind.

C_1 & ξ_1 are constants determined by systems Biot number $\left[B_i = \frac{h r_o}{K}\right)$ using standard lookup table

For centerline temp. ($r = 0$ $R = 0$), so, ~~$J_o = 1$~~.

$J_o(0) = 1$

$$\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_o).$$

Max possible energy transfer:

$$Q_{max} = m C_p (T_i - T_\infty) = \rho V C_p (T_i - T_\infty)$$

$$Q = Q_{max}\left(1 - \frac{2\theta_o^*}{\xi_1} J_1(\xi_1)\right)$$

where, Q is the actual energy transferred upto time t.

$$Q = Q_0\left(1 - \frac{2\theta_0}{\xi_1} J_1(\xi_1)\right)$$

J_1 is first-order bessel fn of the first kind

For a sphere

Governing dimensionless eq^n:

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{2}{R}\frac{\partial \theta}{\partial R} = \frac{\partial \theta}{\partial F_0}$$

$$\theta_{sphere} = C_1 \exp(-\xi_1^2 F_0) \cdot \frac{\sin(\xi_1 R)}{\xi_1 R}$$

C_1 & ξ_1 are constants determined by sphere's Biot number $\left(Bi = \frac{h r_0}{k}\right)$

Instead of bessel functions, the spatial variation in a sphere uses standard sine functions.

Centerline temperature

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_0)$$

$$Q_{max} = m C_p (T_i - T_\infty)$$
$$= \rho \left(\frac{4}{3}\pi r_0^3\right) C_p (T_i - T_\infty)$$

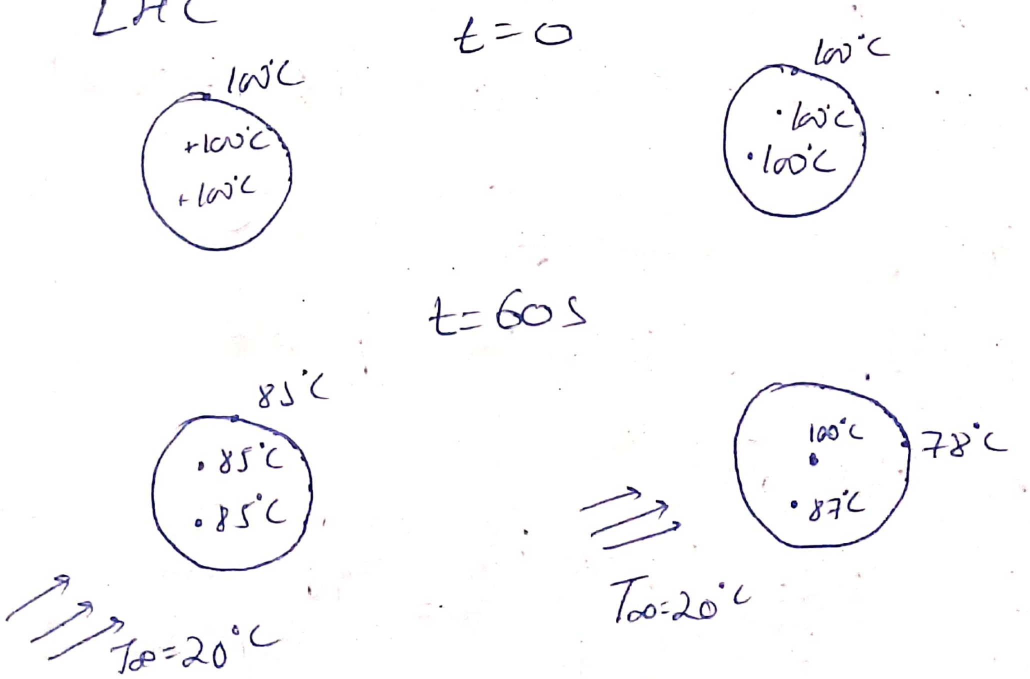
$$\frac{Q}{Q_{max}} = 1 - \frac{3\theta_0}{\xi_1^2}\left(\sin(\xi_1) - \xi_1 \cos(\xi_1)\right)$$

$$Q = Q_{max}\left(1 - \frac{3\theta_0}{\xi_1^2}\left(\sin(\xi_1) - \xi_1 \cos(\xi_1)\right)\right)$$

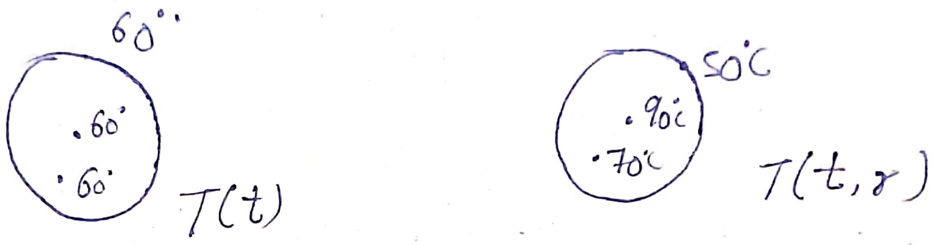
We can only use these approximate equation if $Fo \geq 0.2$.
If it is not we will need more terms other than 1 term approximation.

Let us compare LHC & Actual

LHC | Actual

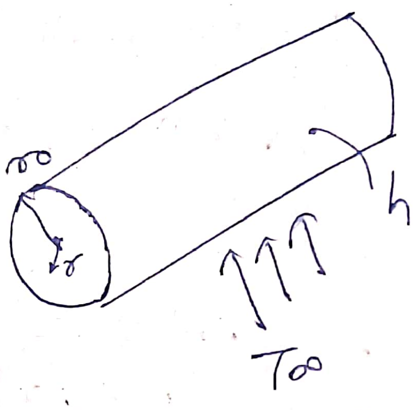


$t = 120s$



We have complicated if $Bi > 0.1$. We can apply LHC if $Bi < 0.1$.

<u>Example</u> a long cylinder ($d = 20cm$) carbon steel.



(a) Find time when center line temp is 800°C

(b) Find surface temp at this time

(c) How much energy transferred in this time.

$T_\infty = 1200°C$ $\quad T_i = 300°C$ $\quad h = 100 \, W/m^2 K$

(Initially it is 300°C at all locations)

Property at $\left(\frac{300+800}{2}\right) = 550°C$

$\rho = 7832$ $\quad k = 51.2$ $\quad C_p = 541$

$\alpha = 1.21 \times 10^{-5}$

$$Bi = \frac{h L_c}{K} = \frac{h \times (r_0/2)}{K} = \frac{100 \times 0.1}{51.2 \times 2}$$

$= 0.097 = \cancel{0.009765}$

< 0.1

Though Bi number is less than 0.1 but it is approx equal to 0.1 so we ~~can~~ will use series solⁿ.

Our eqⁿ is

$$\theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_0)$$

$$F_0 = \frac{\alpha t}{r_0^2}, \quad B_i = 0.097$$

$$\frac{800 - 1200}{300 - 1200} = 0.444 = \theta_0^*$$

Corresponding to Bi number ~~0.0970~~ 0.195

$C_1 = 1.048 \quad \xi_1 = 0.611$

$$0.444 = 1.048 \exp(-(0.611)^2 \times F_0)$$

So, we get $F_0 = 2.30$.

$$\frac{\alpha t}{r_0^2} = 2.30$$

$$t = \frac{2.30 \times (0.1)^2}{1.21 \times 10^{-5}}$$

$$t = \frac{2.3 \times 10^3}{1.21} =$$

In table, value is given when $B_i = \frac{h r_0}{K}$ not $\frac{h(r_0/2)}{K}$

So, ~~in table~~ by table we need value for $B_i = 0.195$

Corresponding to value of $B_i = 0.195$

$C_1 = 1.048$, $\xi_1 = 0.611$

So, $0.4464 = 1.048 \exp(-(0.611)^2 F_o)$

we get $F_o = 2.3$

$$2.3 = \frac{\alpha t}{r_0^2}$$

$$t = \frac{2.3 \times (0.1)^2}{1.21 \times 10^{-5}}$$

$$= \frac{2.3 \times 10^3}{1.21} = 1900 \text{ seconds}$$

Fourier no. for plane wall $= \frac{\alpha t}{L^2}$

infinite cylinder $= \frac{\alpha t}{r_0^2}$

sphere $= \frac{\alpha t}{r_0^2}$

So after 1900 seconds, centerline temp is 800°C.

(b) find surface temperature at this time.

$\Rightarrow$ We use eqn

$$\theta^* = \frac{T(r=r_0) - T_\infty}{T_i - T_\infty} = \theta_0^* J_0(\xi_1 r^*)$$

$r^* = R$ $J_0 \Rightarrow$ bessel function

$= r_0$

$\theta_0^* = 0.444$

$$\theta^* = \frac{T(r=r_0) - T_\infty}{T_i - T_\infty} = 0.444 \times J_0(\xi_1 r^*)$$

~~$\xi_1 r^* = 0.611 \times 0.1 = 0.0611$~~

~~value of $J_0(\xi_1 r^*) = 0.99$~~

$$r^* = \frac{r}{r_0} = \frac{r_0}{r_0} = 1$$

$r^* = 1$ at surface of cylinder

(All variable with * means they are in ratio)

$$\xi_1 r^* = 0.611 \times 1 = 0.611$$

$$J_0(\xi_1 r^*) = 0.91$$

$$\Rightarrow \frac{T(r=r_0) - T_\infty}{T_i - T_\infty} = 0.444 \times 0.91$$

$$\underline{T(r=r_0) = 836\,^\circ C}$$

(c) How much energy is transfered to the cylinder from hot gases from $t=0$ to $t=1900\,s$.

$$\frac{Q}{Q_0} = 1 - \frac{2\theta_0^*}{\xi_1} J_1(\xi_1)$$

$\theta_0^* = 0.444$ $\quad \xi_1 = 0.611$ $\quad J_1(\xi_1) = 0.291$

$$\frac{Q}{Q_0} = 1 - \frac{2 \times 0.444}{0.611} \times J_1(\xi_1)$$

$$\frac{Q}{Q_0} = 1 - 2 \times \frac{0.444}{0.611} \times \cancel{0.287}\ 0.291$$

$$Q_0 = \rho c_p V (T_i - T_\infty)$$

$$= \rho c_p \pi r_0^2 L (T_i - T_\infty)$$

$$Q_0' = \rho c_p \pi r_0^2 (T_i - T_\infty) = Q_0/L$$

$$= 7832 \times 541 \times \pi \times (0.1)^2 \times (300 - 1200)$$

$$= \cancel{2.4 \times 10^9\ J/m}\ 1.2 \times 10^9\ J/m$$

$$Q = \cancel{2.4}\ 1.2 \times 10^9 (0.577)$$

$$= \cancel{1.97 \times 10^9\ J/m}\ \cancel{1.38 \times 10^9\ J/m}\ 7 \times 10^7\ J/m$$

It is the amount of energy transferred from cylinder to hot ~~gasses~~ gases.